Advanced Music Theory

For MUSC 320 at Manhattan University, Spring 2025

Version 1.95 created 4/21/25 by Dave Smey

Introduction

Welcome to our advanced theory text, which I'll roll out in bits and pieces as the semester unfolds. For Advanced Theory we *really* need a custom textbook because we'll be sampling lots of different topics, trying to hit as much as we can. There's no one book that does it all!

As before I'm going to encourage you to invest in some more professional textbooks if you are really interested in this stuff. For the more traditional classical theory topics we'll do first, I recommend:

Kostka, Stefan M., and Byron Almén. *Tonal Harmony*. 9th ed. McGraw-Hill, 2023.

Aldwell, Edward, Carl Schachter, and Allen Cadwallader. *Harmony and Voice Leading*. 5th ed. Cengage Learning, 2018.

(I'm citing the most recent editions, but the smart move is usually to look for an older copy which will be much cheaper. Old editions of most textbooks are perfectly good, but academic publishers revise them in order to disrupt the used book market.)

The "serious" study of rock harmony is actually fairly new, so the work being done on it is still truly theoretical. If one is up for a thought-provoking volume you could try

Doll, Christopher. *Hearing Harmony: Toward a Tonal Theory for the Rock Era*. Ann Arbor: University of Michigan Press, 2017.

Finally, for Modern and "post-tonal" theory I like

Rahn, John. *Basic Atonal Theory.* New York: Schirmer, 1980.

(That's very much out of print, try interlibrary loan), or

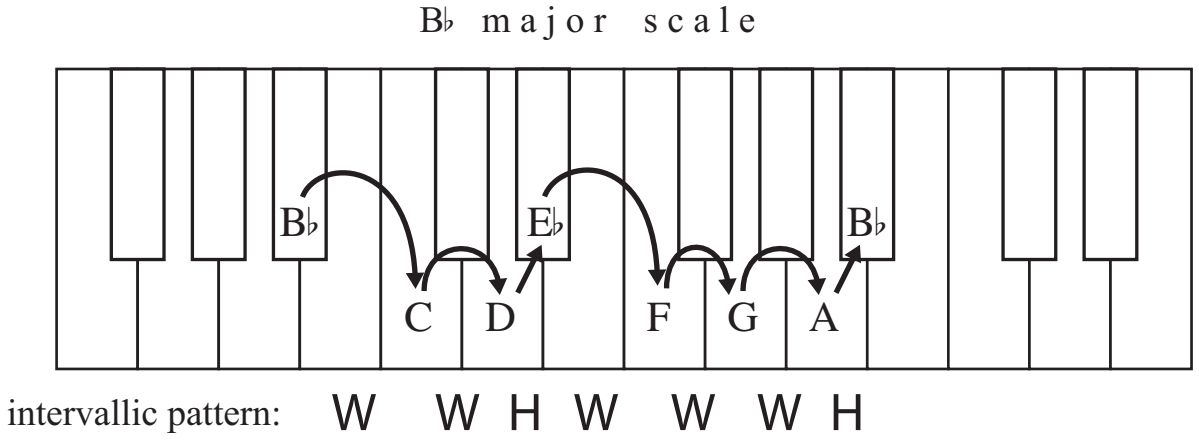
Straus, Joseph N. *Introduction to Post-Tonal Theory.* 4th ed. W. W. Norton & Co., 2016.

Chapter 1: The Basic Elements from Theory I

Here are some diagrams that illustrate basic ideas we learned in Theory I. Remember that if you are still fuzzy on these concepts you can always surf into the MUSC220 blog and look at the Theory I textbook.

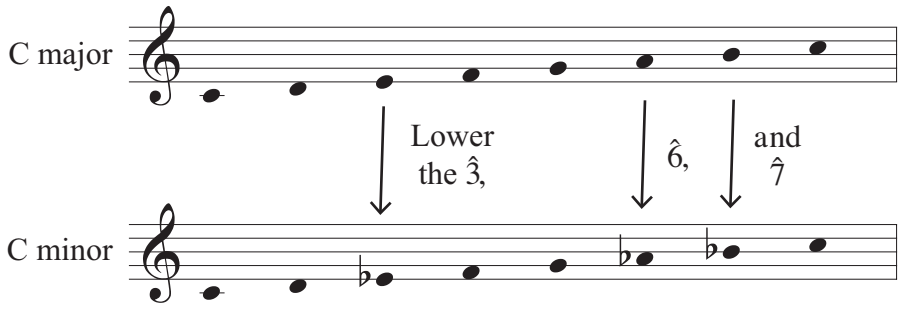
Major Scales

Understanding major scales as WWHWWWH.



Minor Scales

We learned minor as a transformation of major...



...and we also talked about the relative minor / relative major relationship.

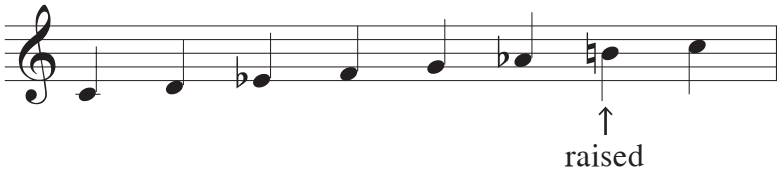
A minor A B C D E F G A

C major C D E F G A B C

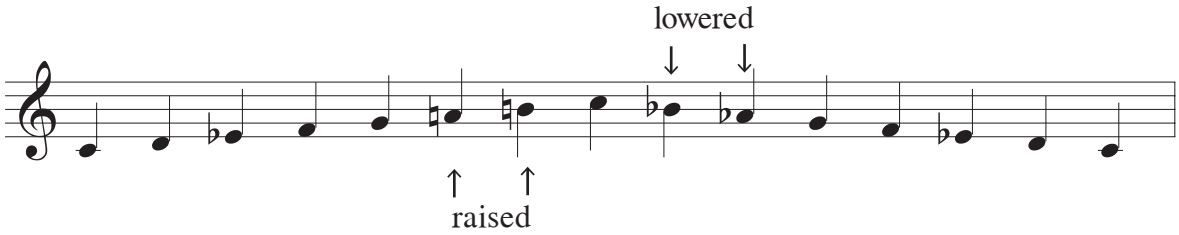
Finding the relative minor.

Variants of minor

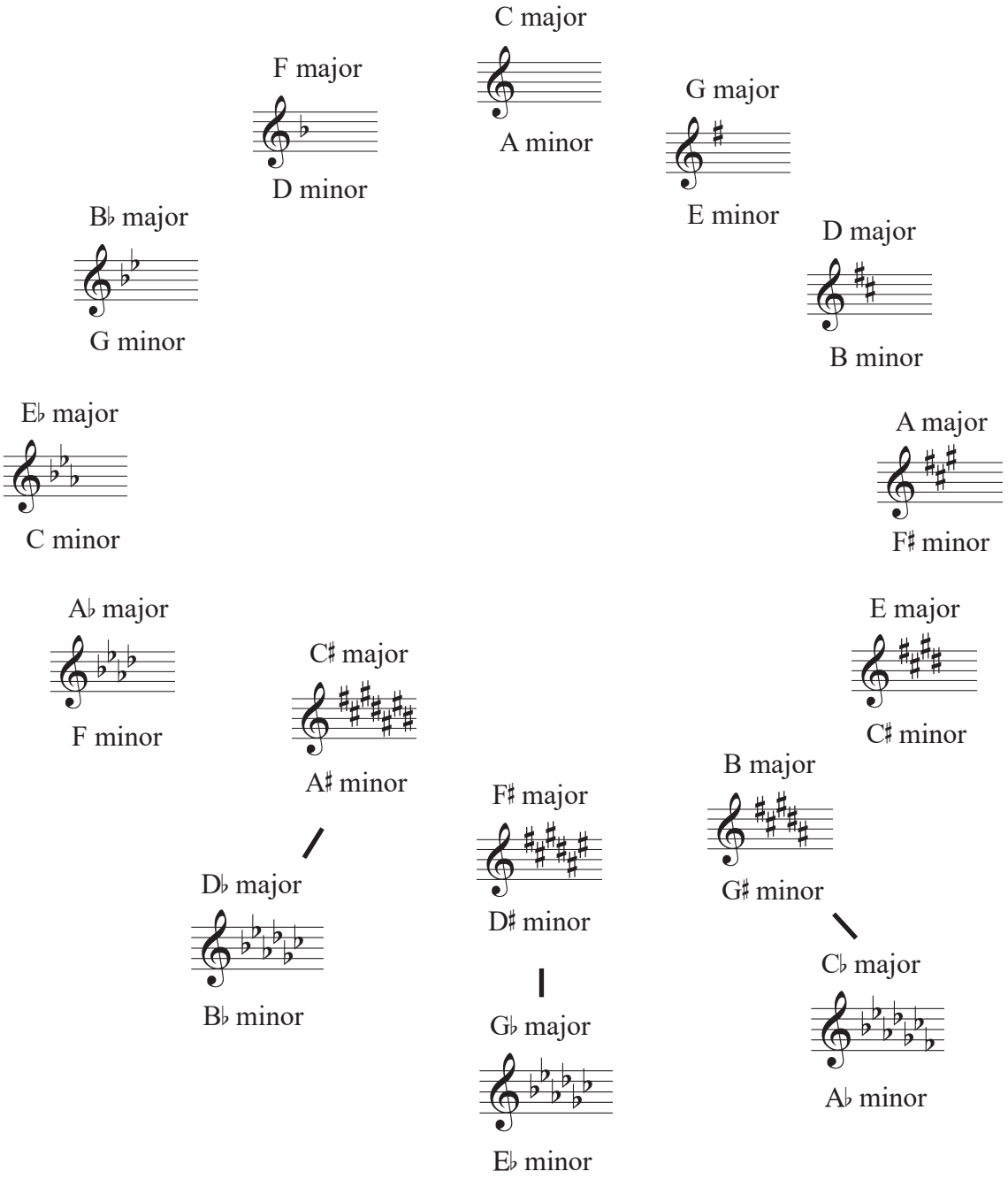
harmonic minor



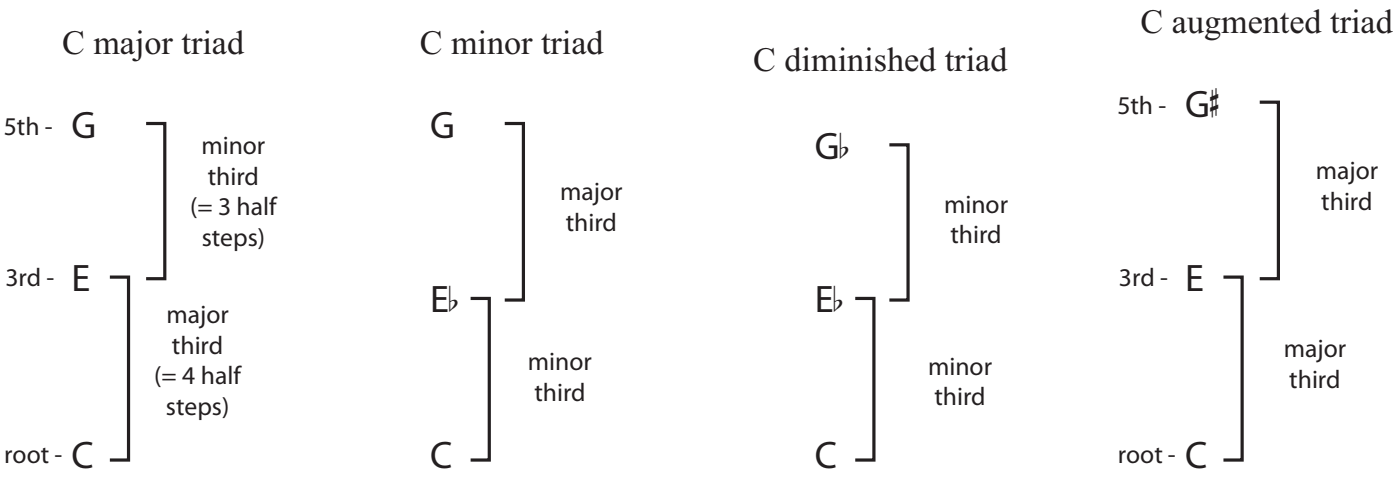
melodic minor



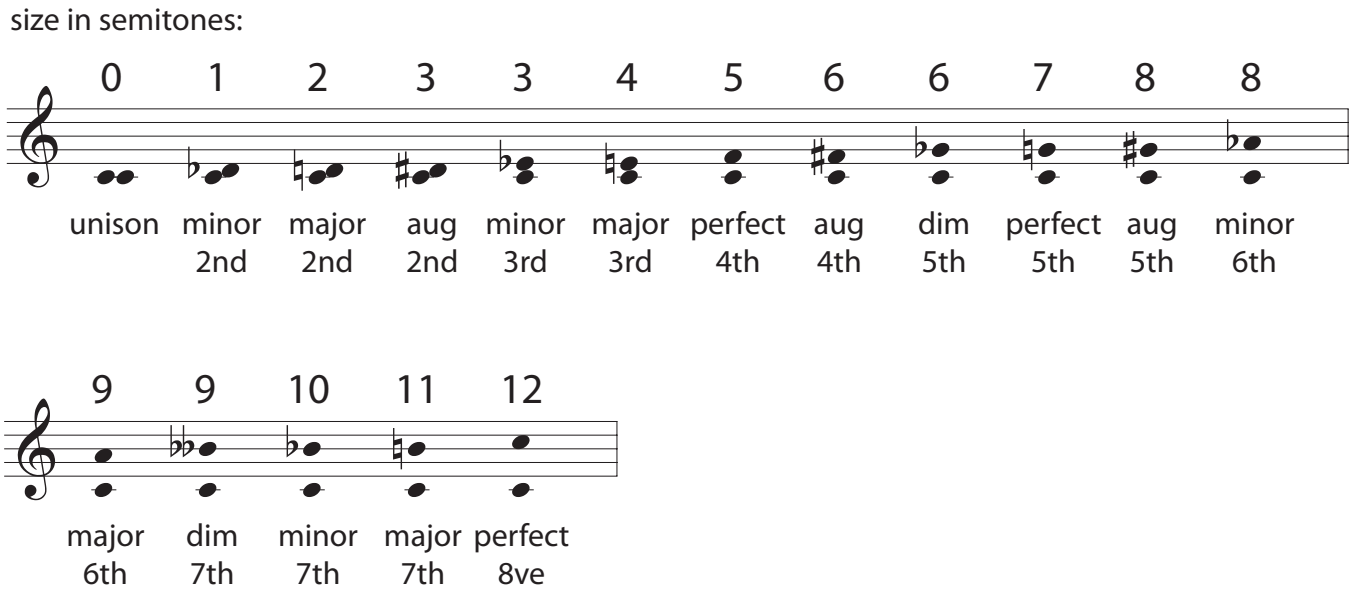
The circle of fifths



The four triad types



Intervals

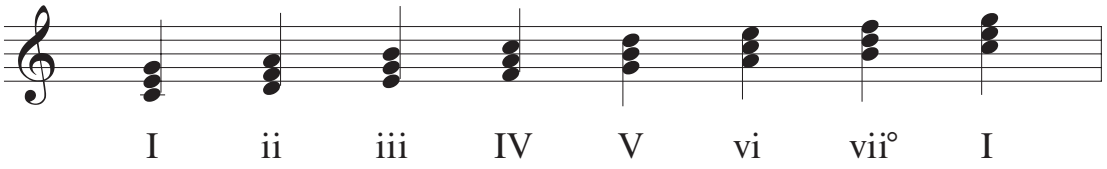


Unisons, fourth, fifths and octaves are the "perfect" intervals. They can be diminished, perfect, or augmented.

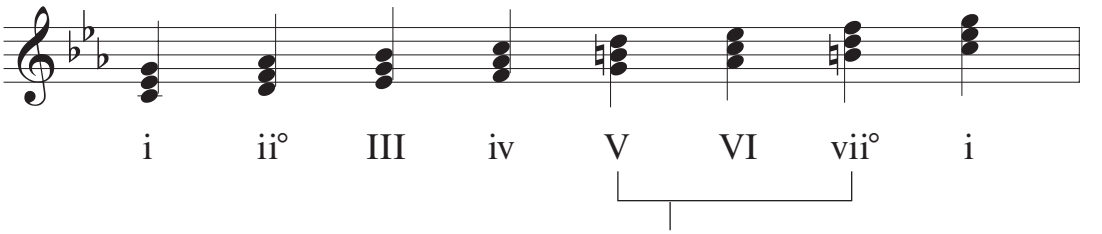
Seconds, thirds, sixths and sevenths are the "imperfect" intervals. They can be diminished, minor, major, or augmented.

Diatonic Triads (a.k.a. the Roman Numerals)

in C major



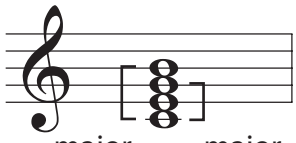
in C minor



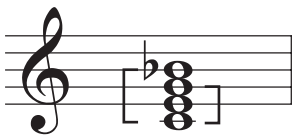
typical Classical-music alterations
with raised leading-tone

Seventh Chords

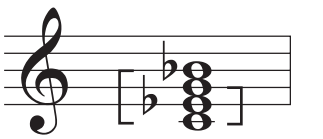
major seventh chord



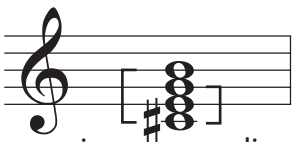
dominant seventh chord



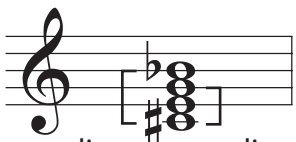
minor seventh chord



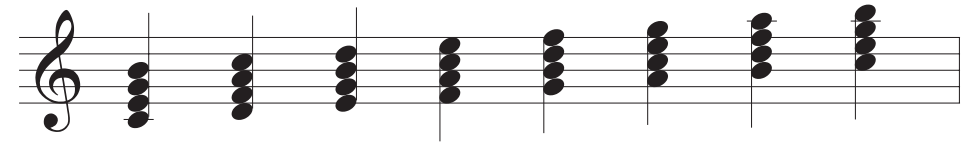
half-diminished seventh chord



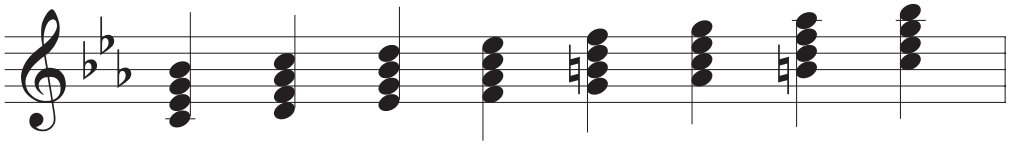
fully-diminished seventh chord



Diatonic Seventh Chords



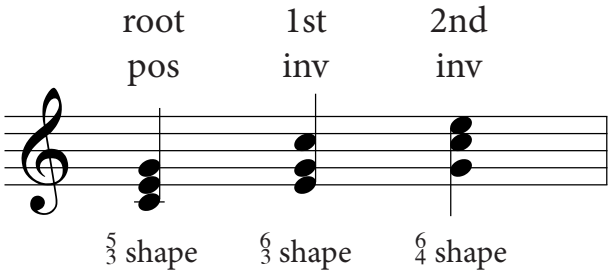
C: I^{M7} ii^{7} iii^{7} IV^{M7} V^{7} vi^{7} $vii^{\varnothing 7}$ I^{M7}



c: i^{7} $ii^{\varnothing 7}$ III^{M7} iv^{7} V^{7} VI^{M7} $vii^{\circ 7}$ i^{7}

Chord Inversions

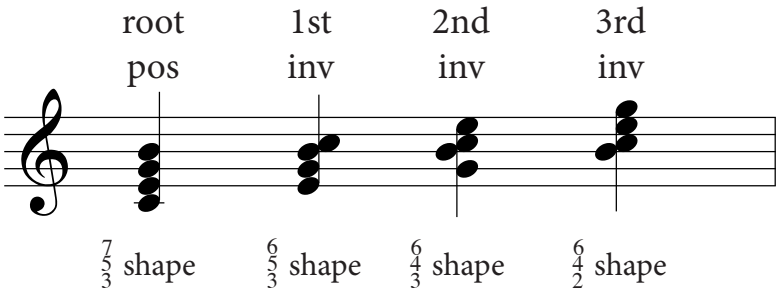
triad inversion shapes



figured bass symbols for "real music"

root position	=	I
1st inversion	=	I^{6}
2nd inversion	=	I^{6}_{4}

seventh-chord inversion shapes



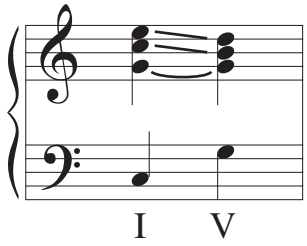
figured bass symbols for "real music"

root position	=	V^7
1st inversion	=	V^6_5
2nd inversion	=	V^4_3
3rd inversion	=	V^4_2

Writing Chord Progressions (with good counterpoint)

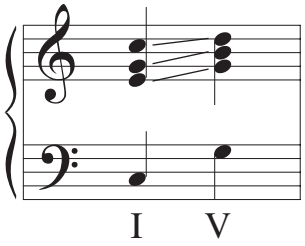
When the bass is moving by **fourth or fifth...**

Common-tone technique



One part holds over, the other two slide into place by step.

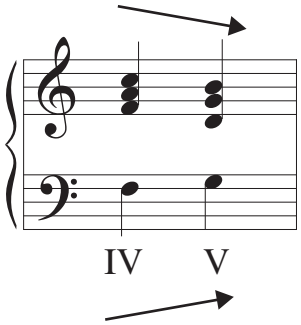
"Next-closest" technique



All parts move by a third or less.

When the bass is moving by **step...**

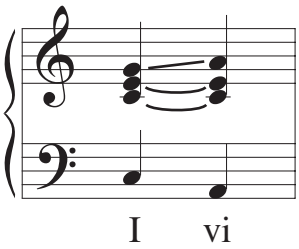
"Step Zone" technique



Slide to the next available shape in the opposite direction.

When the bass is moving by **third or sixth...**

Double common-tone technique

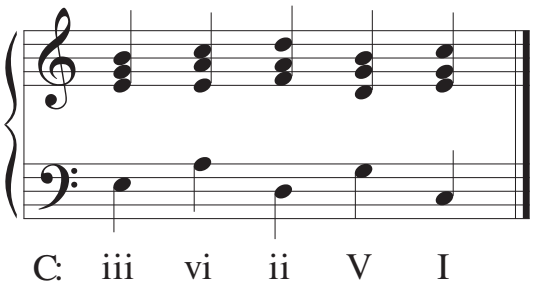


Hold over two notes, remaining tone moves by step.

Chapter 2: Some New Flowchart Details

In our Fundamentals textbook we gradually built up a progressions flowchart as we learned how to make different connections between chords. Let us revisit it now and add a few additional details that pop up frequently in Classical music.

The "spine" of the flowchart is bass notes that fall by fifth. We can start at the ultimate goal, the I chord, think back to V which is obviously a fifth above I, then back further to ii which is a fifth above that. Extending the chain further back ropes in vi and iii. Most people write this kind of progression with a zig-zagging bassline that alternates fourths and fifths.



$$\text{iii} \rightarrow \text{vi} \rightarrow \text{ii} \rightarrow \text{V} \rightarrow \textcircled{\text{I}}$$

In addition, we expanded the position leading up to V, including IV as well as ii.

$$\text{iii} \rightarrow \text{vi} \rightarrow \begin{bmatrix} \text{ii} \\ \text{IV} \end{bmatrix} \rightarrow \text{V} \rightarrow \textcircled{\text{I}}$$

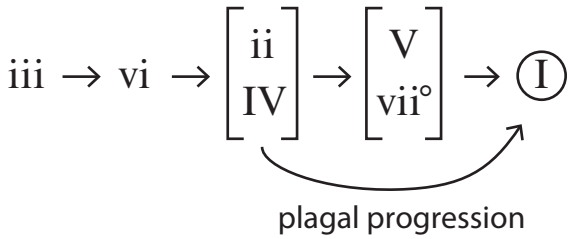
...and even though we haven't worked with it yet, we should include vii° as a possible substitute for V.

$$\text{iii} \rightarrow \text{vi} \rightarrow \begin{bmatrix} \text{ii} \\ \text{IV} \end{bmatrix} \rightarrow \begin{bmatrix} \text{V} \\ \text{vii}^{\circ} \end{bmatrix} \rightarrow \textcircled{\text{I}}$$

Now it is time to add a few exceptions that frequently occur in Classical-type music.

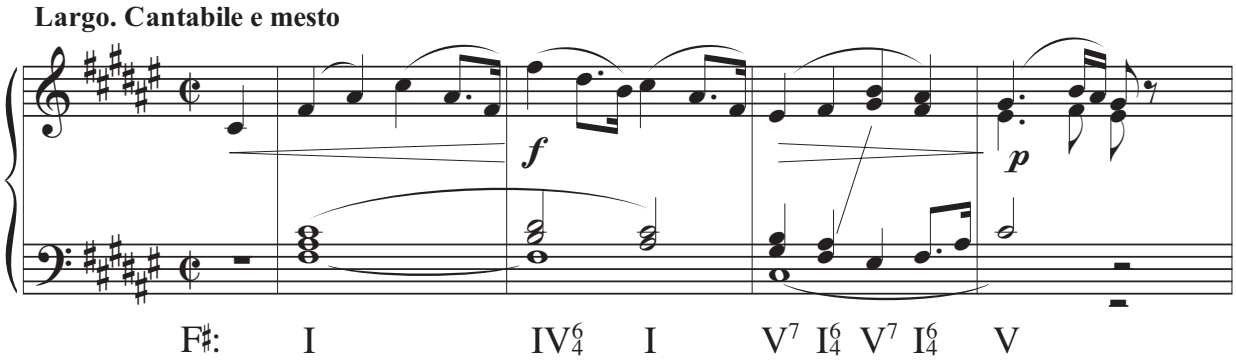
The Plagal Progression

We've talked about this one before. Instead of I-V-I, composers sometimes also like to go I-IV-I. It is the mirror image of the usual progression, going a fifth below tonic instead of above.



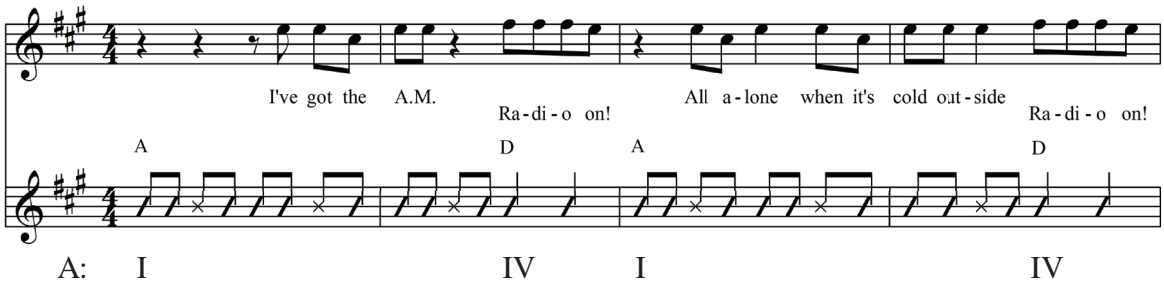
Here's a nice passage from a Haydn string quartet that does it.

Haydn, String Quartet in D major Op. 76 No. 5, second movement mm. 1-4



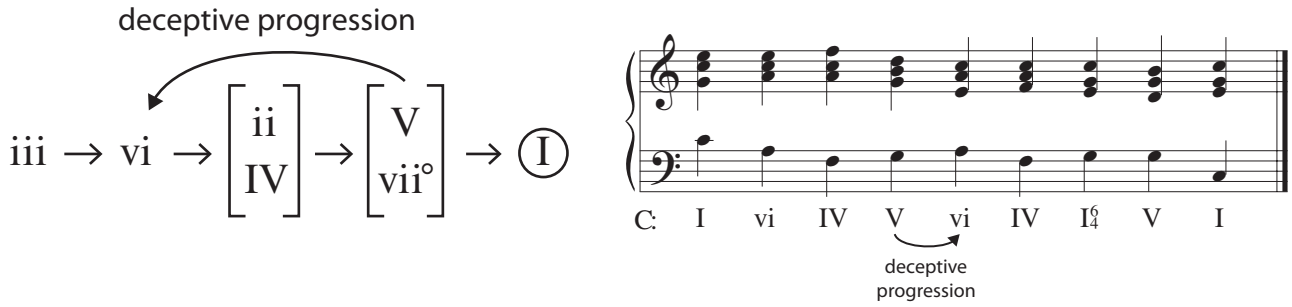
It's also a very popular move in folk and rock music, which seems to generally favor IV to get a "fresher" sound. One notable tune that jams almost exclusively on the plagal progression is "Roadrunner."

The Modern Lovers, "Roadrunner" [1976] ca. 3:23



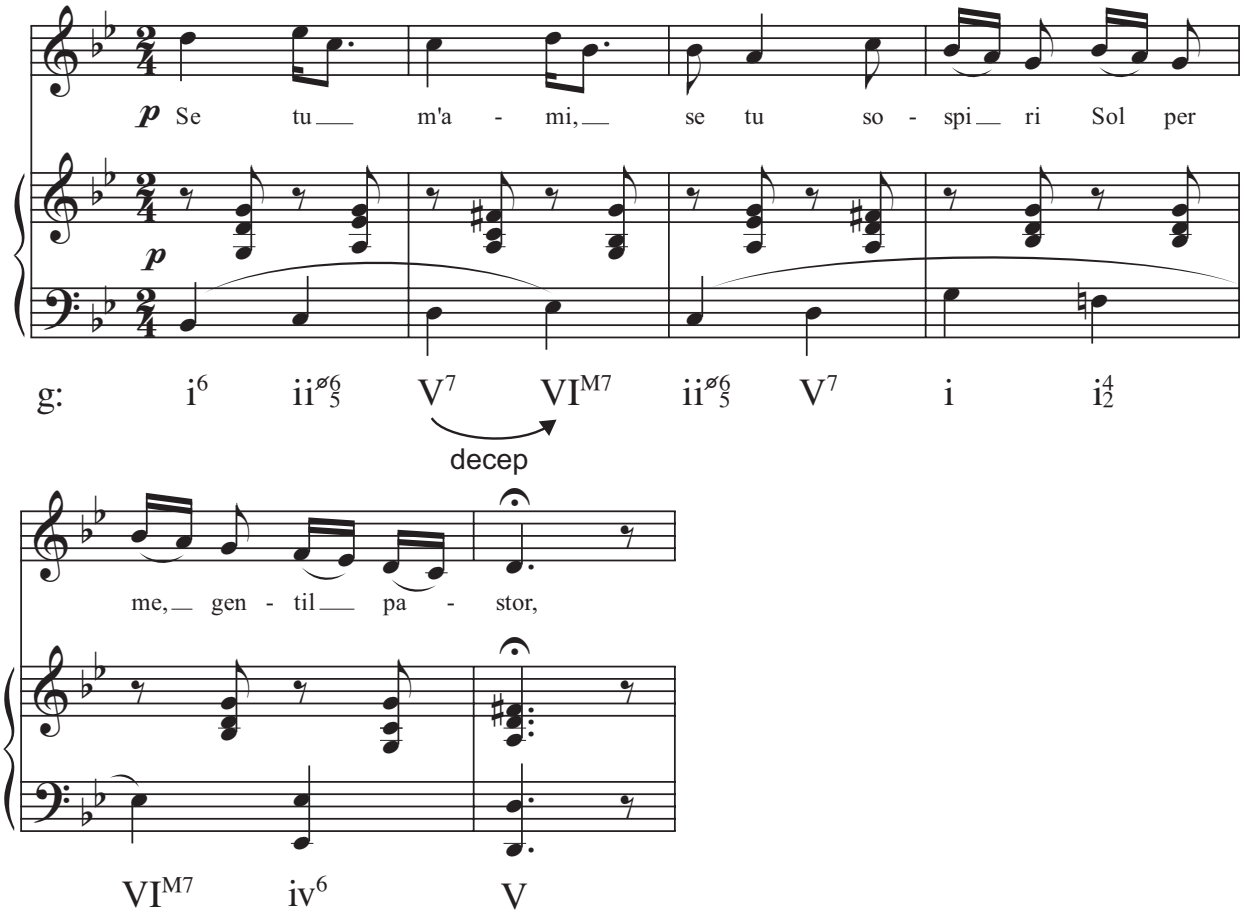
The Deceptive Progression

This is a handy device to extend whatever passage you are writing. You can let your progression proceed to V, but instead of resolving to I as expected you go up to vi. Sometimes this happens at a particularly dramatic moment to create a big "surprise."



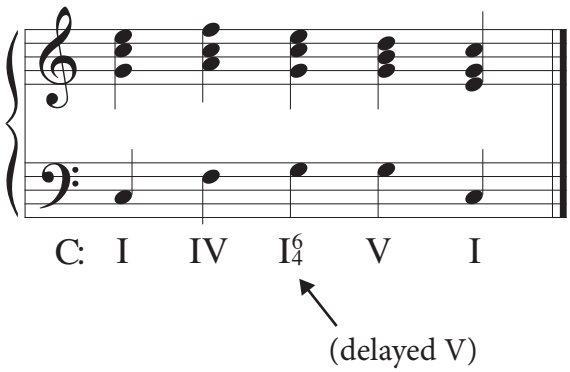
Here is a lovely song from the early 18th century that makes the deceptive move in a minor key.

Parisotti (attrib. Pergolesi), "Se tu m'ami"

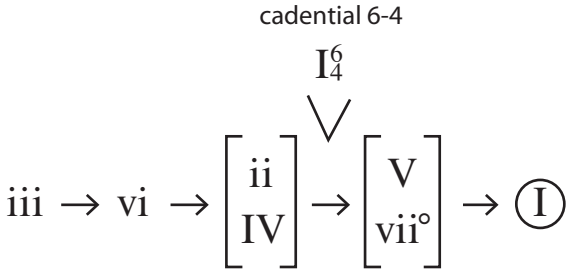


The Cadential 6-4

The cadential $\frac{6}{4}$ is a I chord in second inversion that is inserted before the V. It is a way of delaying the V and making a little more out of it.

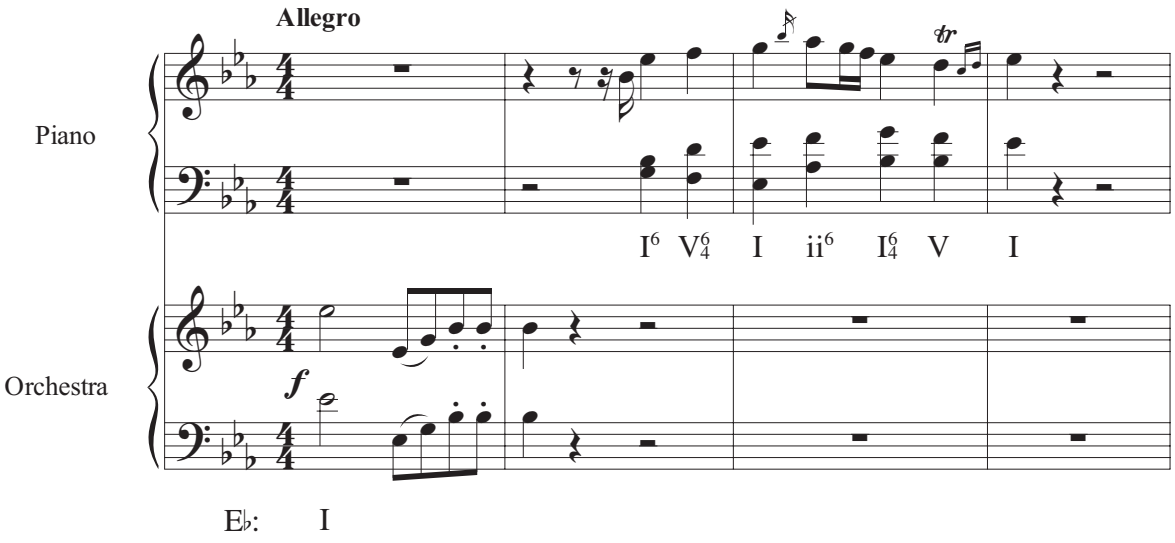


We could imagine it as an optional insertion into our flowchart.



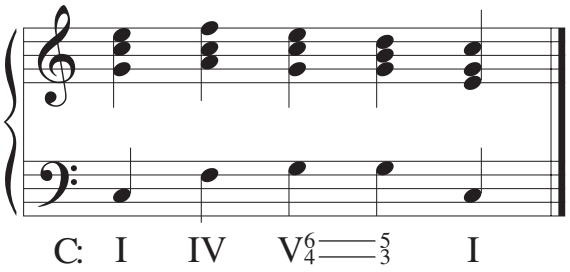
The cadential $\frac{6}{4}$ is very common in Classical music. Here's an example from a Mozart piano concerto, in which the piano repeats a cute little phrase in dialogue with the orchestra.

Mozart, Piano Concerto No. 9 in E-flat major, K. 271 "Jeunehomme," first movement mm. 1-4



Some people call it V^6_4

There is a certain school of thought that labels the cadential 6-4 as V^6_4, not I^6_4. They would analyze our model progression like so:

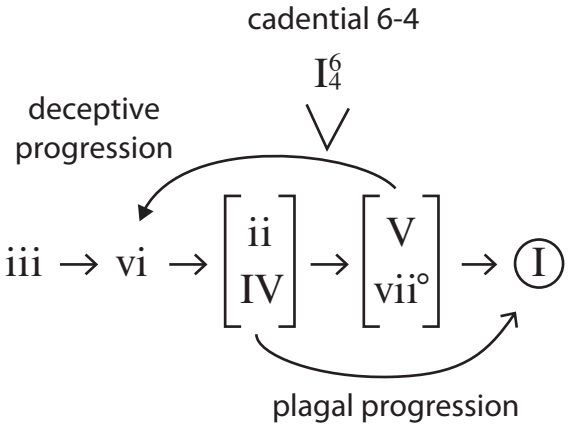


The idea here is that the $^6_4$ is "really" V, and we are just waiting for the tones to fall into place. The little numbers with the lines show that happening. I think this makes sense if you have a lot of experience looking at these little figured-bass numbers, but for the average beginning theory student it is probably nonsensical.

We are not going to do this, and you don't have to worry about it. I'm just warning you that people like this are out there.

Now we've got a full standard diatonic flowchart.

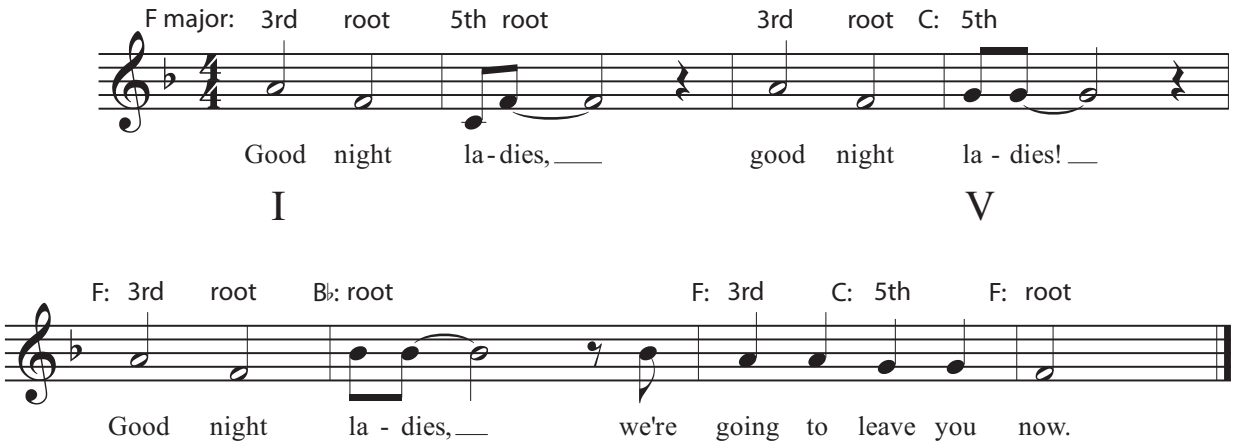
Here it is with all of our new bells and whistles added. The music of Haydn, Mozart, and Beethoven will tend to follow this ordering for a large percentage of the time. This the "diatonic" chart because it is what we expect when a composer is staying strictly in the key and only using notes from the scale. One of the last remaining exceptions is *chromatic* modifications to the progression, using notes from outside the key, which we will consider in Chapter 6.



Chapter 3: Non-Chord Tones

Note: This is mostly just a reprint of Chapter 24 in the Fundamentals textbook. We usually don't get to NCT's in music 220, so it's time to tackle them now.

Most of the music you hear every day is based on chord progressions. A composer chooses a sequence of chords, and the notes in the progression provide a sort of backbone or framework for the music. It is possible to make an entire melody out of selected notes from the chords, like with this familiar folk tune:



However, we usually need more notes to flesh out the music. We add decorations and even interesting distortions to our harmonies in order to make it all sound a little more alive. These extra, added notes are called *non-chord tones*.

In order to illustrate the various kinds of NCTs I'm going to use a few graphic symbols:

● = chord tone (consonant, stable)

○ = non-chord tone (dissonant, subservient to more stable note)

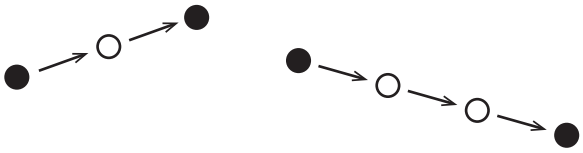
↗ = stepwise connection

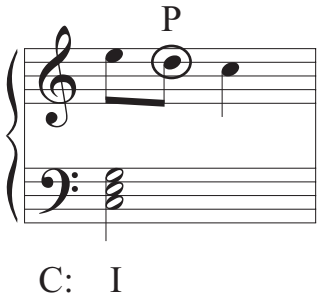
Basic melodic NCTs

First, let's consider some fairly simple ways to elaborate a musical line.

Passing tone

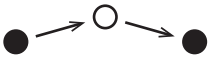
A passing tone comes from a note and continues on to a different note.

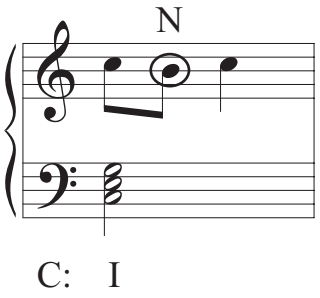




Neighbor tone

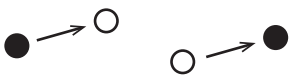
A neighbor tone goes back to the same note it came from.

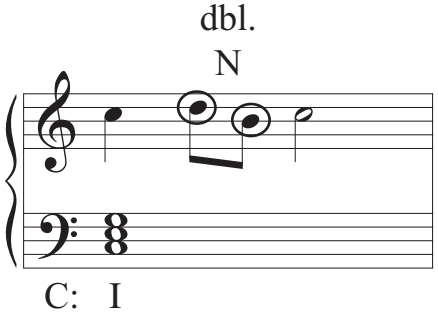




Double neighbor

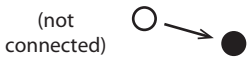
Instead of going immediately back to the note it came from, a double neighbor figure makes an "above, below, then back" pattern (or the opposite.)

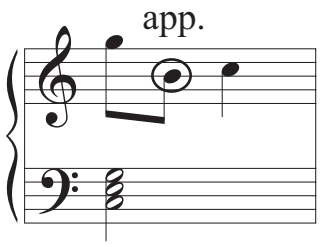


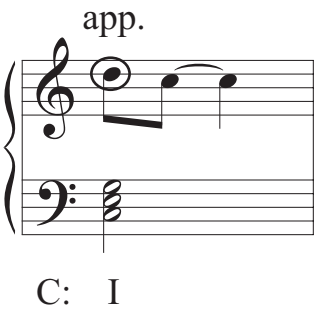


Appoggiatura [from the Italian word "to lean"]

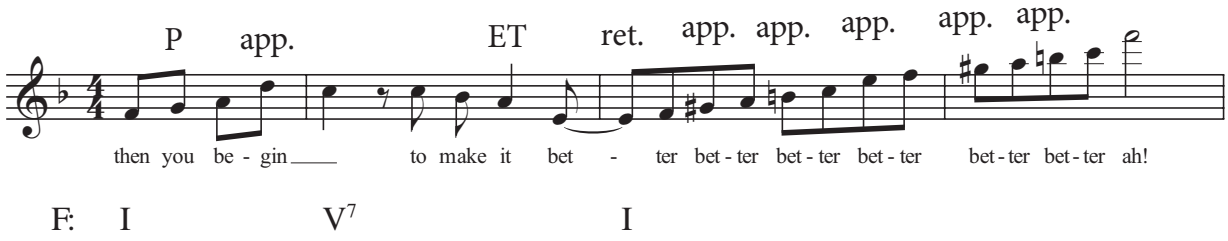
Also known as an "incomplete neighbor." An appoggiatura resolves to a chord tone, but it is approached by leap or just out of the blue.





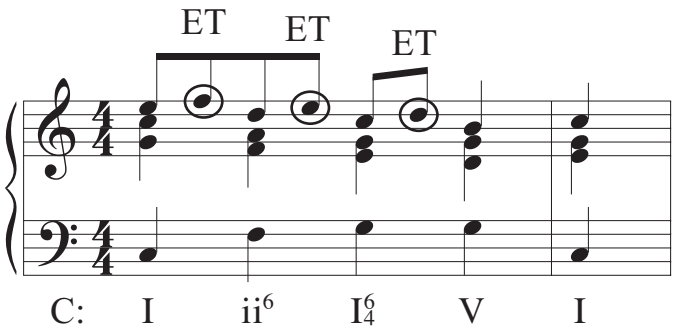


One of my favorite examples of this figure in pop music occurs towards the end of The Beatles' "Hey Jude," as Paul McCartney methodically climbs up the I chord for two octaves, decorating each chord tone with an appoggiatura.



Escape Tone

The escape tone is the only NCT that doesn't resolve to another chord tone - instead it comes *from* a chord tone. It's the opposite of the appoggiatura. The most common use of ET's in classical music is to decorate a scalar ascent or descent, like the example on the right.

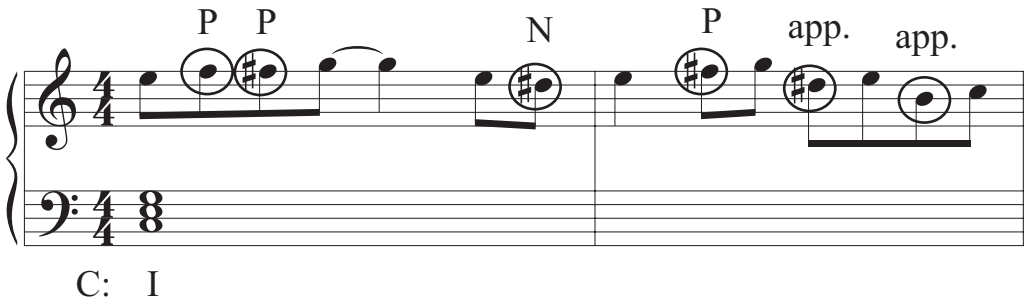


A few interesting distinctions

Diatonic vs. Chromatic

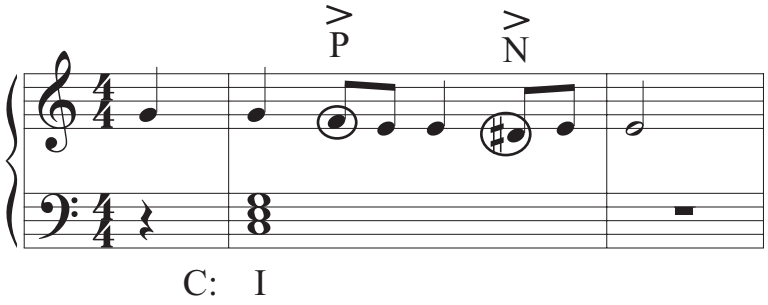
Diatonic NCTs use tones that belong to the scale. Chromatic NCTs use tones that are outside the scale (and require accidentals.)

A passage with mostly chromatic non-chord tones



Accented vs. Unaccented

An unaccented non-chord tone will be relatively "weak" and "unimportant" compared to the notes around it. An accented NCT, however, will stick out - it falls on the beat or it is longer than surrounding notes. You can mark accented P's, N's, or app's with a little accent mark.

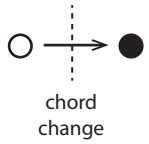


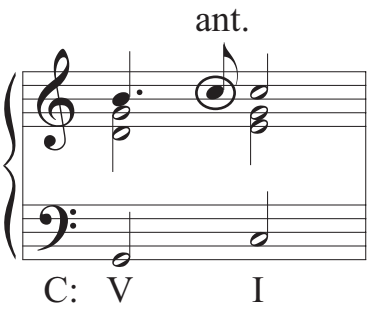
Displacement NCTs - ant., sus., ret., pedal

We also have a few NCTs that involve notes being where they don't belong - they either arrive early or hang on late.

Anticipation

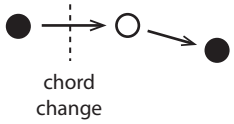
Here a note simply comes in early instead of waiting for the next chord. All of these displacement NCTs can involve either a sustained note (which is held through more than one harmony) or a repeated note. Anticipations are frequently a repeated note.



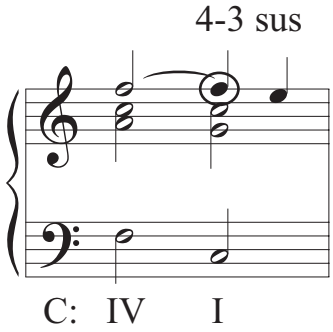


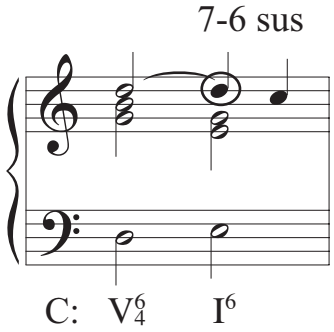
Suspension

Suspensions are perhaps the coolest NCT, but they also involve the most complicated theoretical baggage. Simply put, a suspension hangs over from the previous harmony and then *resolves down by step* into the new harmony.

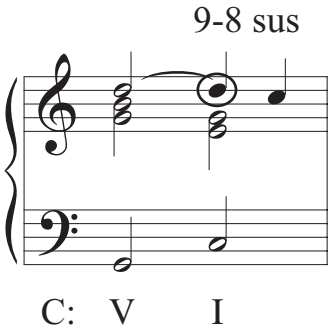


For some reason theorists like to classify suspensions with figured bass numbers. (We aren't really going to worry about it, but you might see technical discussion like this somewhere in the future.) Remember that figured bass numbers are all about the interval above the bass - when you figure out your suspension labels you don't have to think about what the roman numeral is, or the key - just measure the interval above the actual bass note. If you really want to understand this, you could look carefully at these examples and see if you can understand where the numbers come from.

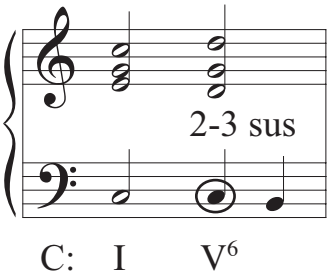




This one is always called "9-8" (even though we might be tempted to say "2-1.")

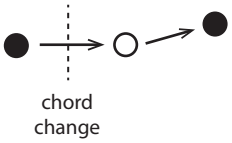


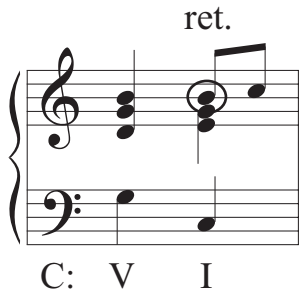
A suspension in the bass is always called "2-3." It refers to the fact that somewhere in the chord there is a tone that the bass is dissonantly rubbing up against - it makes a second against the bass, but once it resolves it makes a consonant 3.



Retardation

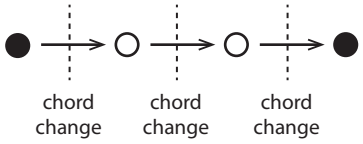
If a held-over note doesn't resolve down, it isn't a suspension. It's a retardation. Retardations are less common than suspensions, and they usually don't get the fancy figured-bass style labels.

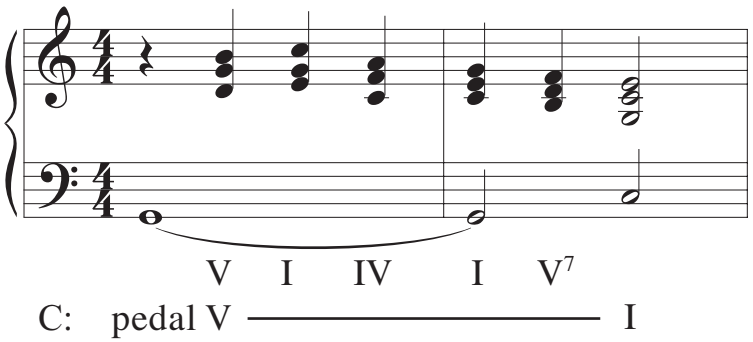




Pedal Tone

A pedal tone is held across a number of harmonies. It is usually in the bass, and usually on scale-degrees $\hat{1}$ or $\hat{5}$. The harmonies on top usually go away from and then back to consonance with the pedal tone. You can analyze it in two layers - on the bottom you can indicate "Pedal V" (or "Pedal I"), and then in an upper layer you can track the harmonies that happen over the bass (with no inversion symbols.)





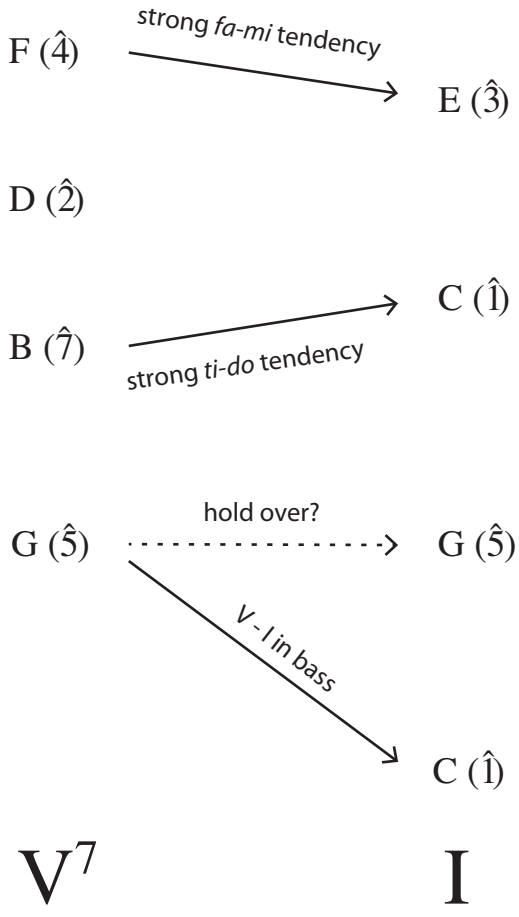
Chapter 4: Progressions with V7

The most common seventh chord in most kinds of music is V^7, which usually wants to resolve to I. In this chapter I will show you two quick formulas to write progressions with this slightly spicy harmony.

Tendency Tones

There is often a sense that the tones of a V^7 chord "want" to resolve in a certain way. This is probably caused by a combination of cultural and perceptual factors.

Let us consider a V7 chord in C major:



As it resolves, the seventh of the V^7 (*fa*) "wants" to resolve down to the third of the I (*mi*). The leading-tone (*ti,* the third of the chord) "wants" to go up to *do*. (There is something about the closeness of the target note, only a half-step away, that seems to make these resolutions particularly satisfying.)

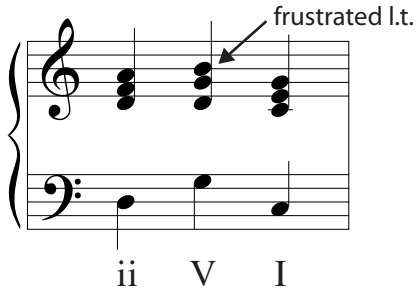
What the remaining tones want to do is more of a practical matter than a perceptually urgent one. The fifth of the chord (*re*) should probably go to *do*, but it could easily go up to *mi* if it needs to.

The root of the V7 (*sol*) probably wants to make a strong *sol-do* bass line. However, if it is in an upper voice it can be a common tone that holds over into I.

The Frustrated Leading-Tone

Theorists are particularly concerned with the tendency of the leading tone, which wants to go from *ti* to *do*. If the leading tone is in the top line of your V or V^7 chord, it must go to *do* or else it is called a "frustrated leading tone."

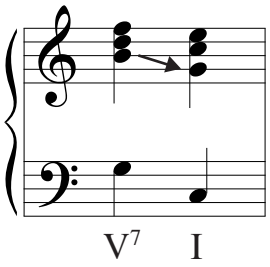
This is true even with a plain V chord! The idea is that leaping away from the leading tone is disappointing, because we anticipate the conventional resolution to *do* and will be annoyed if we don't get it.



A plain V chord with frustrated leading tone.

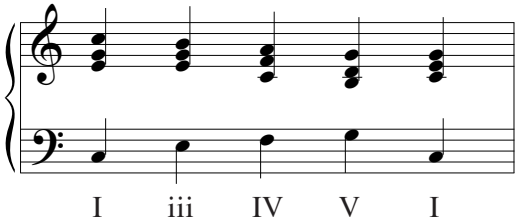
However, this rule is actually pretty narrow - it only applies when the leading tone is exposed in an outer voice (soprano or bass) and you leap away from it instead.

If you hide the leading tone in an inner voice it's considered fine. This will actually be one of our strategies to make good V^7-I's.

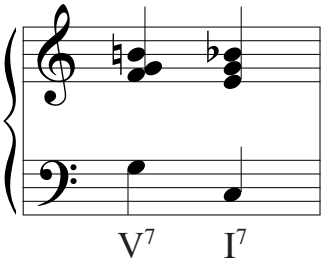


Frustrated leading tone in inner voice (which is fine)

There are also a few situations where the leading tone might connect downward by step that are not at all disappointing.



In this progression, the leading tone is part of the iii chord. It continues down by step and I don't think anybody would object. Stepwise connections are strong, so the motion from C-B-A is compelling and not disappointing in any way.

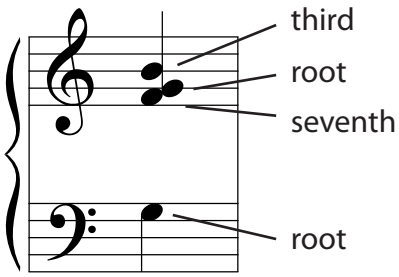


Similarly, in contexts where we go from one dominant seventh to another, the third of one chord often sinks down to the seventh of the next. Let us imagine that we are going from a V^7 to a I^7 in some kind of bluesy progression. This is also fine.

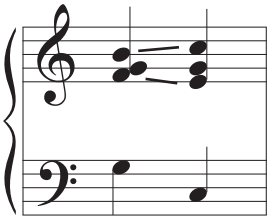
So with all of this tendency-tone business out of the way, let's look at two quick strategies to make good V^7-I progressions.

Strategy #1 - Omit the fifth, double the root

Surprisingly, you don't need to use all of the notes in your dominant seventh chord. You can omit the fifth, and nobody will miss it.

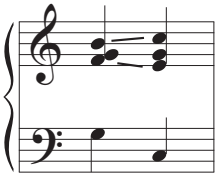


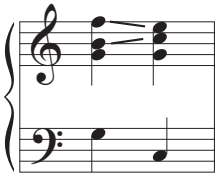
With this technique you build V^7 with the root on the bottom and root, third, and seventh on top.

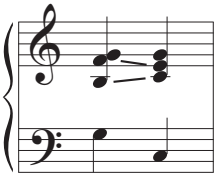


After that you follow all of your tendency tones. *Ti* goes to *do*, *fa* goes to *mi*, and that extra *sol* in the upper voices holds over as your common tone. This will make a normal-looking I chord.

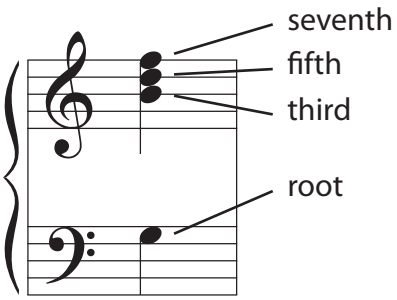
There are a few ways to rotate around your upper tones. I like the formation I've been using above because it fits very comfortably under the hand, but there are two other possible options as well.



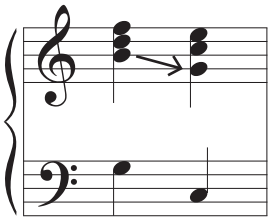




Strategy #2 - Frustrated leading tone in an inner voice



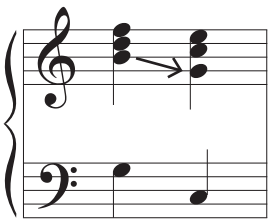
Here we will use all four tones of the V^7. The root is in the bass, everything else is on top, and you should make sure the leading tone (i.e. the third of the V^7) is somewhere in an inner voice.

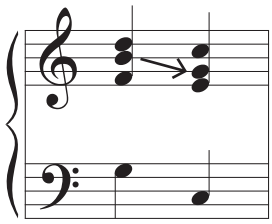


The leading tone ducks down to *sol*. Everything else slides into place.

Because *ti* is in an inner voice, it is not considered "frustrated." (You might say it is frustrated but nobody notices, so life goes on. It may have to do a little therapy later to deal with any lingering feelings of disappointment.)

Because the third of the chord cannot be on top, there are only two rotations of this voice-leading strategy we can use.

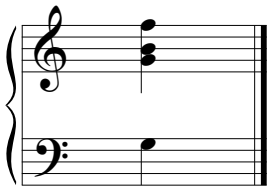




Simple I-V^7-I: Mirroring your I chord

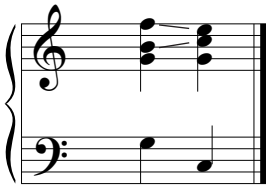
To keep things simple we are going to write a lot of I-V^7-I progressions. You can write these in three easy steps. (1) Plan your V^7 and your "strategy." (2) Resolve to I. (3) Copy your final I as your first I as well. This will always work!

Here I am planning a V^7 with missing fifth.



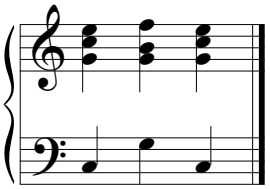
C: I V^7 I

We resolve it like always.



C: I V^7 I

Copy the last chord as the first.

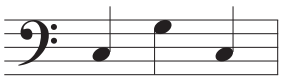


C: I V^7 I

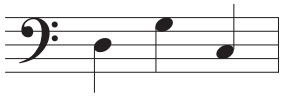
Other ways to get into your V^7

It's pretty hard to get into trouble as you enter into your V7 as long as you don't jump too much and remember your step zone. Let's consider three likely progressions that will contain V7.

I - V^7 - I and ii - V^7 - I



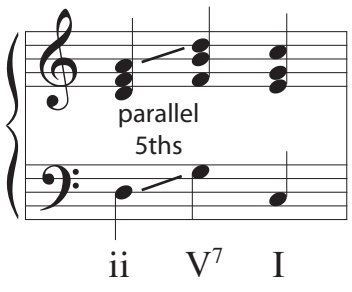
I V^7 I



ii V^7 I

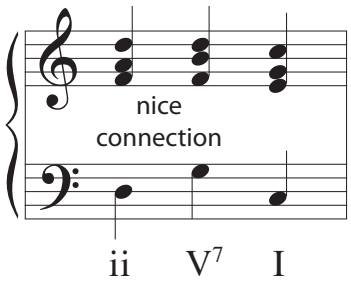
With either of these, the bass is jumping into the V7 by fourth or fifth, so we can use connections that are similar to our old common-tone and next-closest techniques. Remember that the point of these techniques was always to avoid jumping too much in your upper voices.

Here's an example of a progression gone wrong because the upper voice jumps by a fifth.

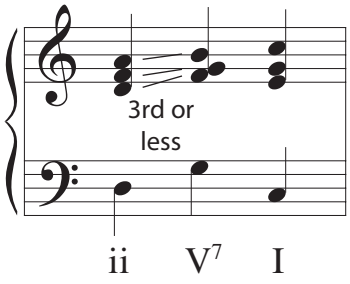


ii V^7 I

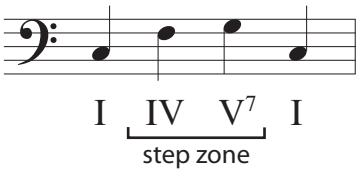
If we move our first chord up we get a nice, smooth connection with two common tones.



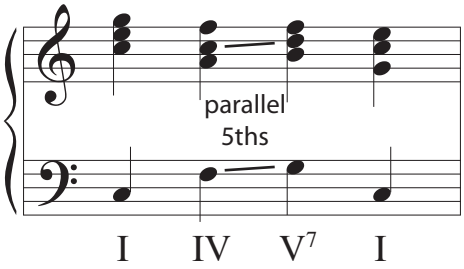
Here is one more ii-V^7 connection that moves a little more. Everything still moves by a third or less, using the same logic as our old "next-closest" connection.



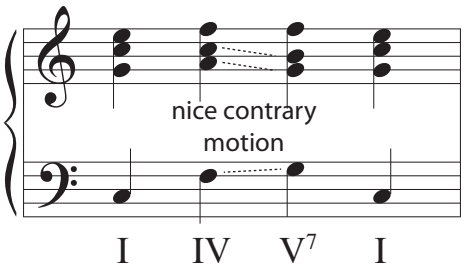
I - IV - V^7 - I



Here there is a step zone from IV to V, so you want to make most of the upper voices go in the opposite direction.



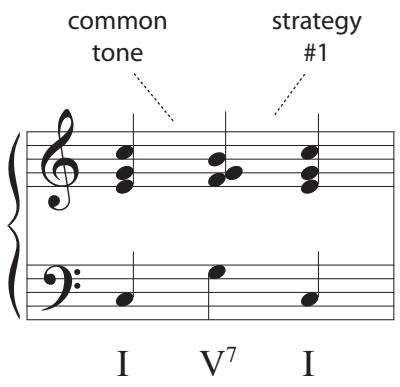
If you don't, you might make bad parallels against the bass.

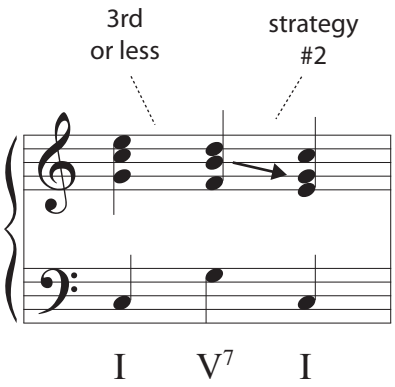


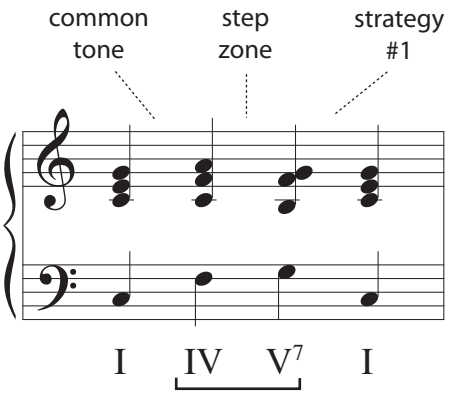
...so we want to pull most (or all) of the parts downward, in contrary motion. Here we hold over the F and move down the other two.

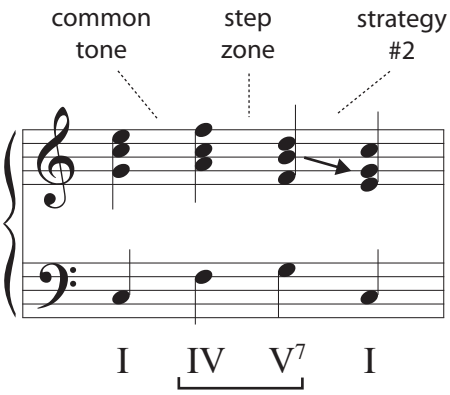
Overall, if you have a good feel for our old triad techniques and you learn our two new strategies for building and resolving V^7 it should be easy to write these more interesting progressions and avoid problems.

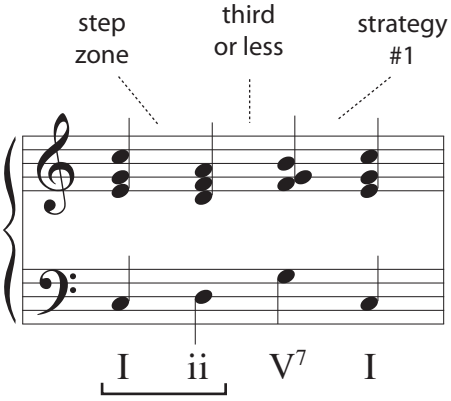
Let's conclude this section with annotated examples of various possible progressions that include V^7. This is not an exhaustive listing by any means.

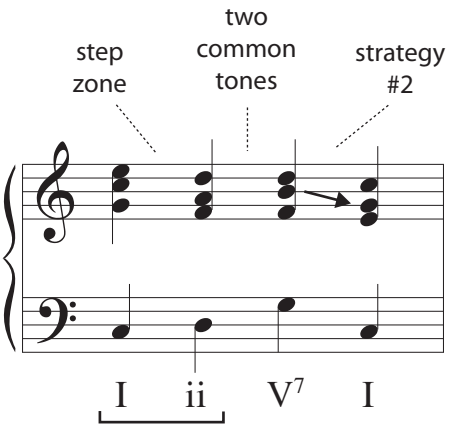






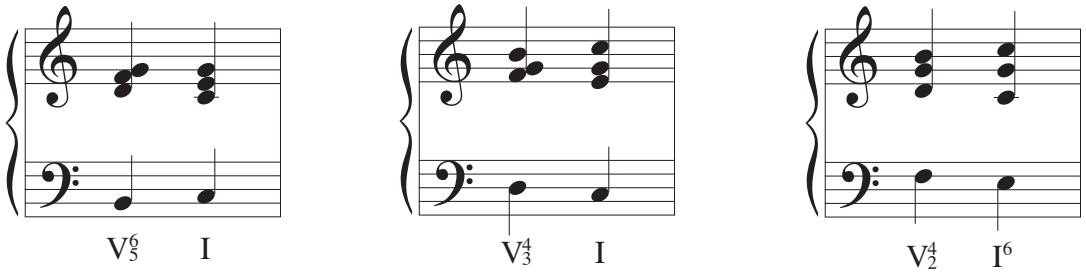






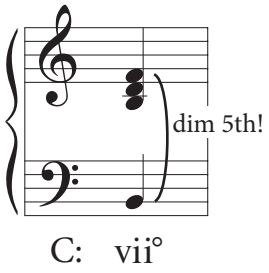
V^6_5, V^4_3 and V^4_2

Inverted V^7's are usually pretty straightforward. You want to use all four notes in the chord - put whatever tone you need in the bass and distribute the others on top. Then, just follow your tendency tones. *Fa* goes to *mi*, *ti* goes to *do*, *re* should probably also go to *do*, and *sol* holds over.

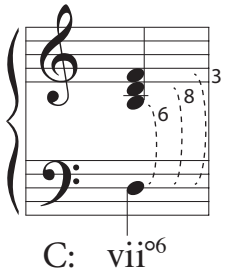


Chapter 5: Playing with vii°7

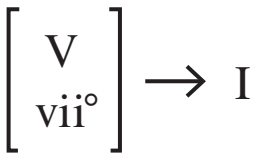
Up until now we've completely avoided the vii° chord, and there's a pretty good reason for that. Classical composers didn't like the sound of the vii° in root position, because they thought the diminished fifth was too harsh against the bass.

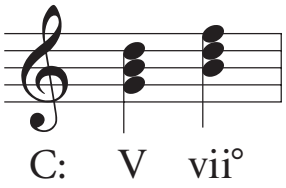


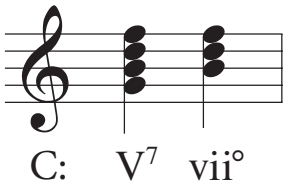
Interestingly, they didn't have a problem with vii°6, because putting the chord in first inversion makes all "nice" intervals against the bass.



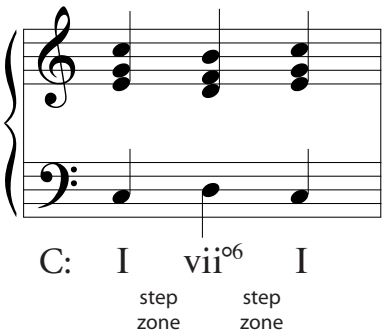
The vii°6 triad usually functions as a substitute for the V chord. If you compare the V and vii° you can see that they share two notes in common. And, perhaps more convincingly, the vii° is equivalent to the upper tones of the V7 chord.







If you really wanted to write a I - vii°6 - I progression you could pull it off by thinking of your step zones.

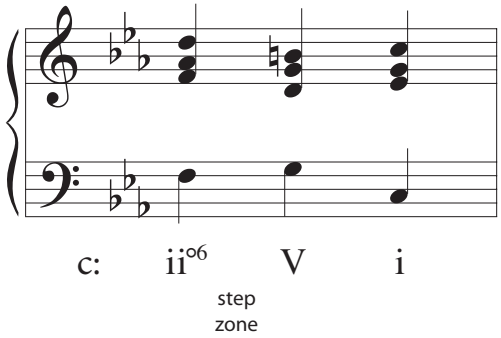


However, we aren't really going to bother with that! Just be mindful that this is something you might see in classical music.

ii°6 in minor

Also the same principle holds for ii° in minor. Classical composers were unlikely to use the diminished ii° in root-position, instead preferring ii°6. Instead of ii° - V - i they would write ii°6 - V - i.

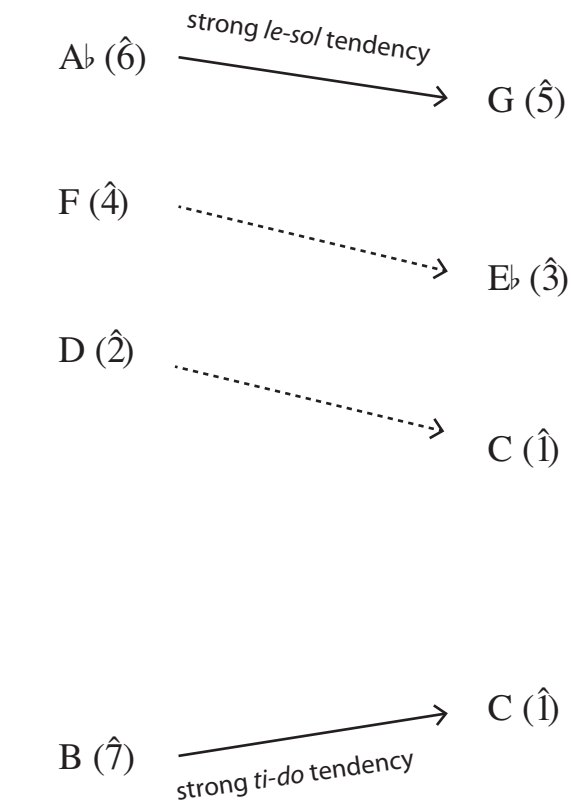
This is actually easy to write if you remember to mind the new "step zone" in the bass line.



But again I don't think we need to worry about this particular progression.

vii°7 in minor

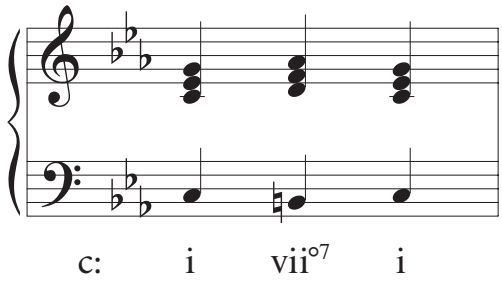
What we WILL practice a little bit is vii°7 to i in minor. Like with V7, the tones in this seventh chord have certain "tendencies" that we need to observe. As long as we follow the tendencies it will be a piece of cake.

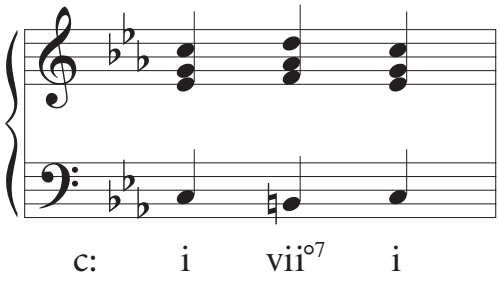


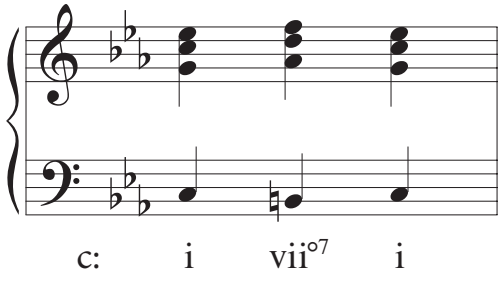
c: vii°7 i

Our root of the vii°7 wants to resolve to the tonic, *ti-do*. We'll take the upper tones and slide them down to the i chord. In particular, the seventh of the vii°7 has a very strong tendency to sink downward, *le-sol*.

To keep it simple we'll do i - vii°7 - i, not worrying about other chords that might lead up to vii°7.

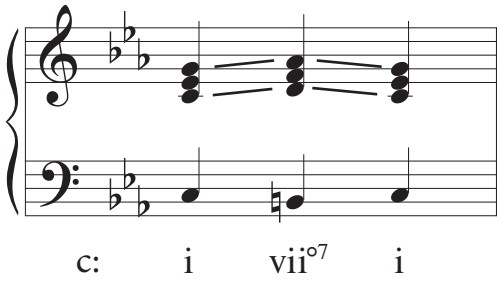






Unequal fifths

Perhaps the most eagle-eyed reader might notice something odd about our first i - vii^{o7} - i progression. We don't normally push triad shapes up and down like that. Isn't it making parallel fifths in the soprano and tenor?



The answer is no, because these are "unequal fifths." C-G is a perfect fifth but D-A♭ is diminished. It's not literally parallel motion so this is considered fine. You are only allowed to get away with this in your upper parts, though.

Chapter 6: Chromatic Harmonies
Part I: Secondary Dominants

Up until this point we've dealt exclusively with *diatonic* harmonies, chords that can be derived from the scale that the key is based on. Diatonic chord progressions have a certain conventional sound to them, because they make patterns that we are very familiar with and the notes all fit together in a predictable way.

However, sometimes a composer wants a little more juice from their progression, and so they turn to *chromatic* harmonies, chords that use notes from outside of the scale. It's time to deal with some chromatic possibilities that we might encounter in different kinds of music.

Using and abusing our Roman numerals

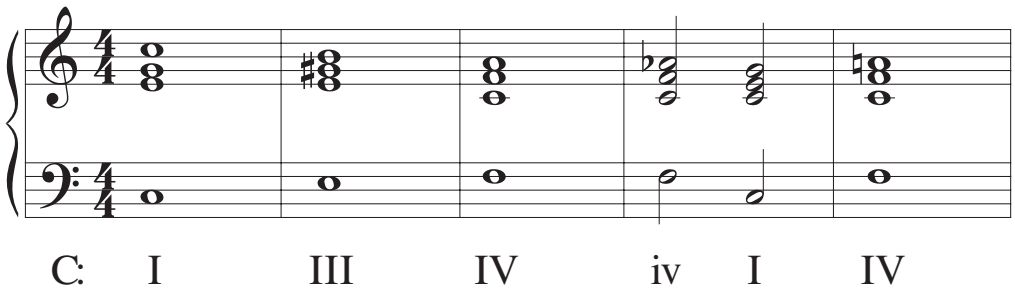
Our roman numerals are probably adequate to ID any triad or seventh chord that might pop up in a progression. For instance, let's look at this passage from David Bowie's "Space Oddity" [1969]. I'll give the lyrics with chords over them, like you might see on the internet.

```
C                                E
This is Ground Control to Major Tom

                        F
You've really made the grade

        Fm              C                          F
And the papers want to know whose shirts you wear

         Fm                 C                      F
Now it's time to leave the capsule if you dare...
```

These chords don't fit together into any key we know about! C sounds like the tonic here, so we can assume that the whole passage is in C major and describe the chords as I, III, IV and iv. I and IV are diatonic, but III and iv are chromatic.

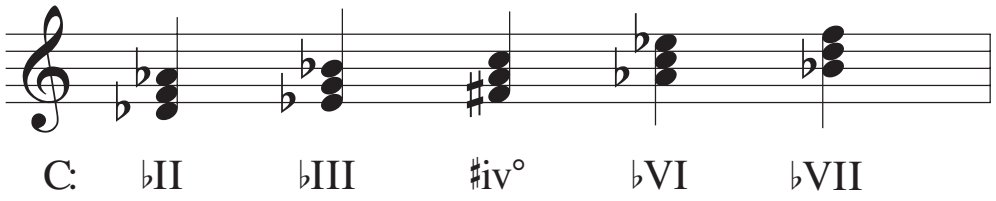
Putting roman numerals on these chords is actually pretty straightforward - the E chord is major so we made it a capital III, and as the F chord turns minor it gets a lowercase iv.

I'll write a theory class-style keyboard progression that you could play while you sing the tune. Note how we need extra accidentals to make the chromatic chords.



Putting flats and sharps in front of the chord symbols

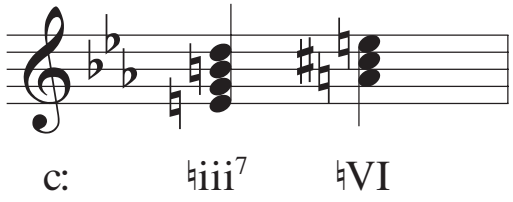
Sometimes a chord will appear in a position that doesn't normally have a roman numeral, because the root is outside of the scale. In these situations we can flat or sharp our chord symbol to indicate that it falls a half step lower or higher than we'd otherwise expect. Here are some examples of chromatic chords in C major with flatted or sharped roots.



Chromatic Roman numerals in minor

In minor I don't think we should be so quick to sharp our roman numerals. A lot of the diatonic harmonies in minor are already "flattened," in a sense, if we think of them relative to major. III in minor is a half step lower than iii in major, VI is lower than vi, etc.

If you see a three or six in minor that is a half step higher than normal, you could use a natural sign to show that.

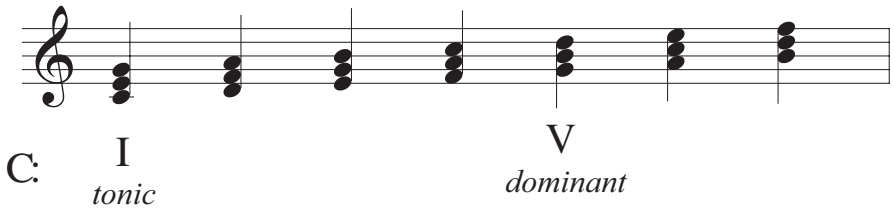


Wouldn't it be nice...

...if this was the whole lesson on chromatic harmonies. Adapting our roman numerals to cover any chord that might pop up is a simple enough concept! However, there are certain chord relationships that are traditionally described in a somewhat more complicated way, and we are going to learn how to use this terminology as well. So, let us now turn to the world of *secondary dominants*.

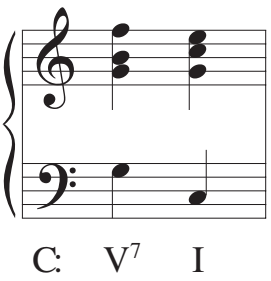
Secondary Dominants

First, we need to do a bit of review. The I and V in any key are also frequently referred to as the *tonic* and *dominant*.



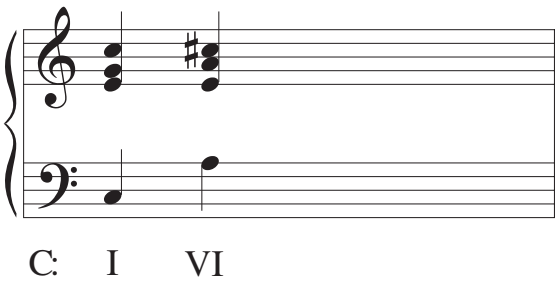
We know that the dominant is an extremely important harmony, since V-I is a common progression that helps clarify what key were are in.

Also, the seventh-chord type we've learned as the "dominant seventh" (i.e. a major triad with an added minor seventh) is so-called because it usually appears in this same position in the key. V^7-I is just as common and important as V-I.

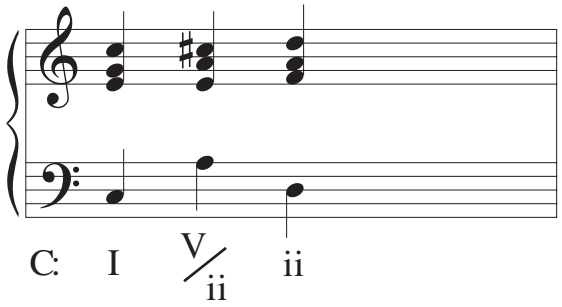


Now, imagine you are learning a piece of music and you encounter a harmony that is either **major when you wouldn't normally expect it**, or **a dominant seventh when you wouldn't expect it**.

As an example let's create a progression that starts on I in C major, but then continues to a major VI.

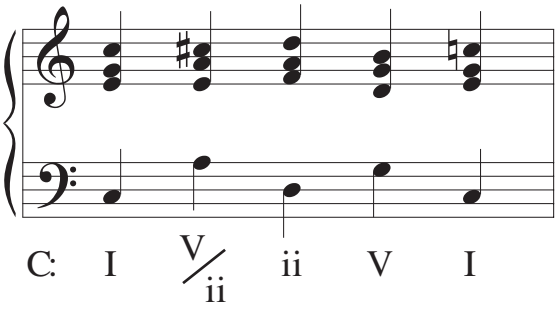


You should inspect this chord and see if it continues to another whose root is a fifth down. We will make our VI proceed to d minor, the regular ii, which is indeed a fifth down from A.

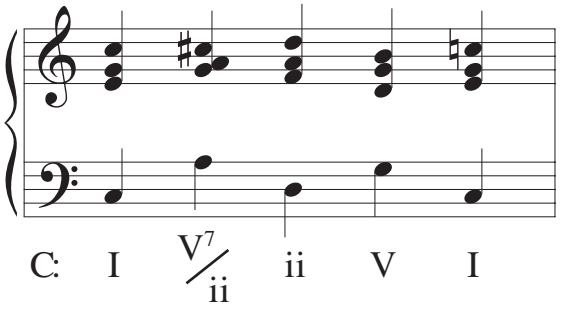


What we are seeing here is a *secondary dominant*, "V of ii." The idea is that the d minor chord briefly gets its own V (as it would appear in the key of d minor), and the progression proceeds in a miniature dominant-tonic motion. In our analysis we mark it as "V / ii" with a slash notation.

Then we can keep going in the key of C major, with ii proceeding to V and I. Our chromatic harmony gave us a brief twist which helped us push out to ii, creating a neat C-C#-D line in the top part.

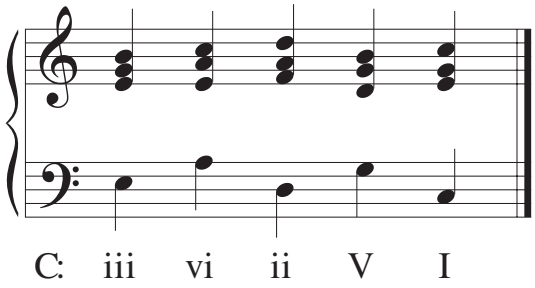


It would also be possible to make our chromatic chord "V^7 of ii," like so:



The "target" is the next chord in the chain.

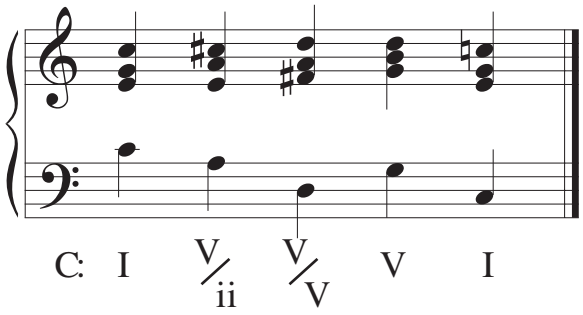
The hardest aspect of analyzing secondary dominants is keeping track of this "V of" language, to figure out what the target of the chord would be. I think it helps to think about the flowchart of typical diatonic chord progressions, which is built around a series of falling fifths.



iii → vi → ii → V → (I)

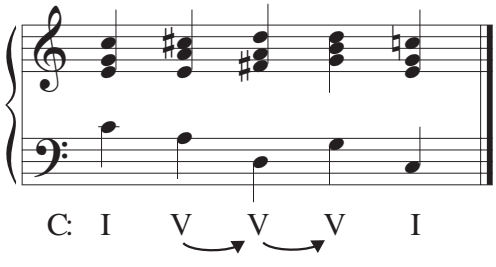
If you remember that iii leads to vi, vi leads to ii, and ii leads to V, it is easy to decode many secondary dominants. A major triad on III will be "V of vi" (i.e. V of the next thing in the chain). A major triad on VI is "V of ii." The target is the next chord in the regular sequence.

Naming the secondary dominants after their targets gets particularly awkward when multiple dominants happen in a sequence, a phenomenon that occasionally occurs in real music.



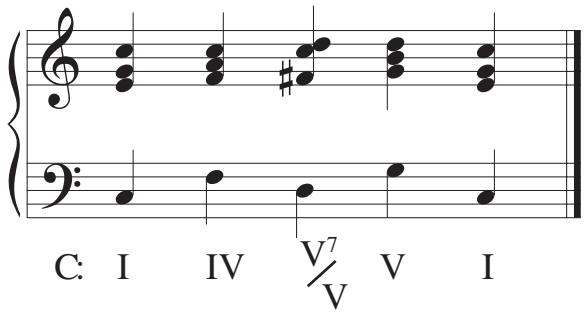
A nice alternative to "V of ____"

One slightly less fussy way to notate this in your analyses is to use an arrow that points forward to the target.

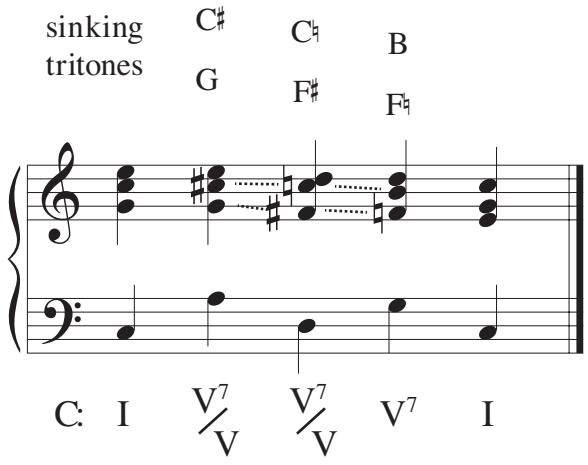


Secondary V^7's

It's also possible to make these secondary dominants with V7 chords, not just triads. Here's a progression that switches from IV to V^7/V on the way to V. Secondary dominant sevenths are actually very common in Classical-type music.



When dominant sevenths chain together in a sequence the tendency tones often work a little differently than usual. Usually we would expect the third of a V7 to resolve up, like a typical leading-tone *ti-do*. However, when two V7's connect the two "juicy" parts of the chord will slide downwards - third connects to seventh and vice-versa. The two tones make a tritone that continues to sink by half-step.

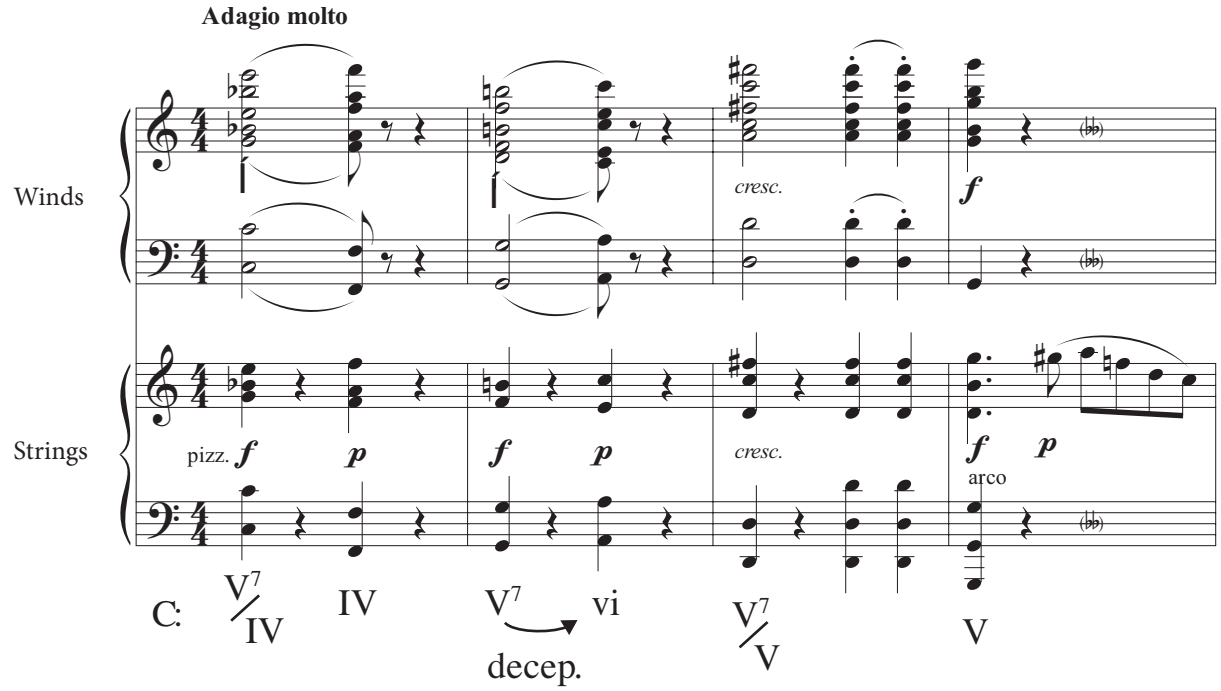


V^7 of IV

V^7 of IV is a particularly interesting case because it is "really" the I chord, which has been turned into a dominant seventh. While the I is usually stable and at rest, V^7 of IV introduces extra dissonance that pushes away from the tonic and jump-starts a new progression.

Beethoven's First Symphony famously starts with a V^7 of IV, followed by a deceptive progression and a cadence on V (which he will eventually have to "undo" as he settles into C major.) This was a bold and intentionally disorienting opening to one of his first large-scale works.

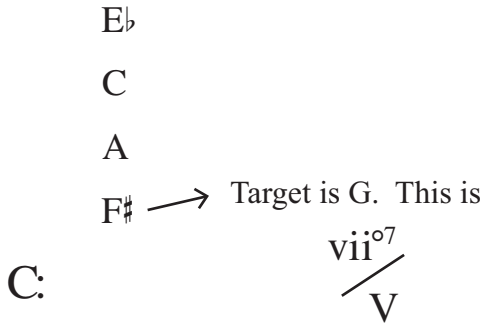
Beethoven, Symphony No. 1 in C major Op. 21, first movement mm. 1-4



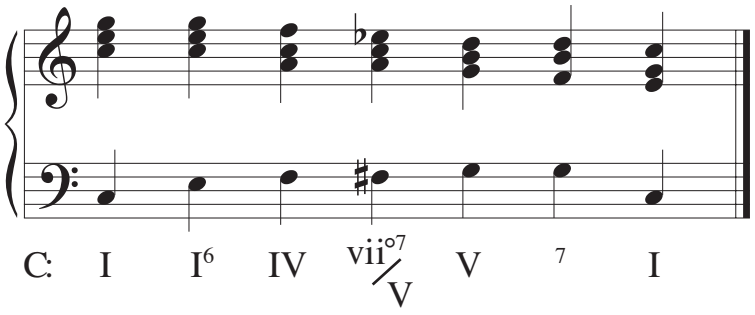
Secondary vii°7

It is also possible to insert the vii°7 of any target chord into the progression. This harmony will usually be built on a root that is not in the key, so you can't say that it is "really" a more typical chord that has been altered. Instead it will usually appear in positions that lie in the cracks between diatonic notes to create cool chromatic motion.

If you find a fully-diminished seventh chord in your progression where it would not normally be, **its target is a half step above the root**.

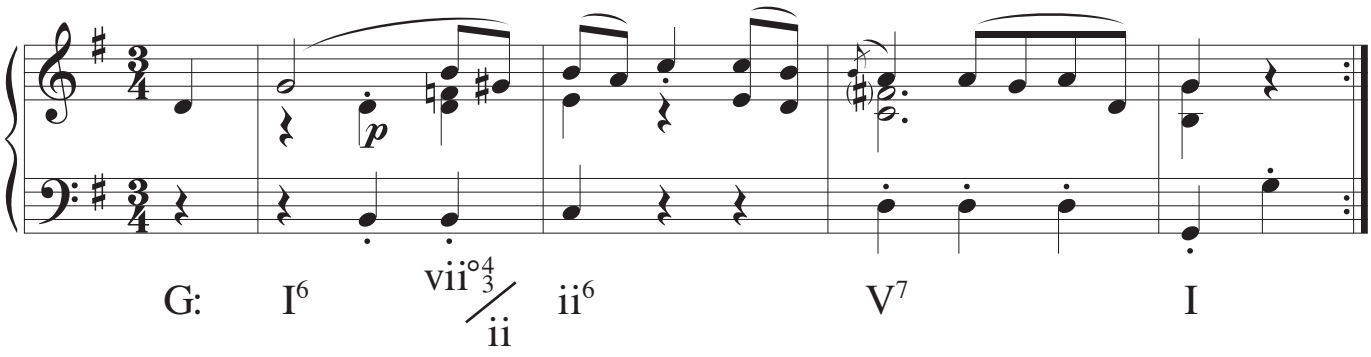


Let's insert our vii°7/V between IV and V to create some chromatic movement.



Here Haydn inserts an inverted vii°7 of ii on his way up to V7.

Haydn, String Quartet in G Major Op. 64 No. 4, second movement, mm. 5-8

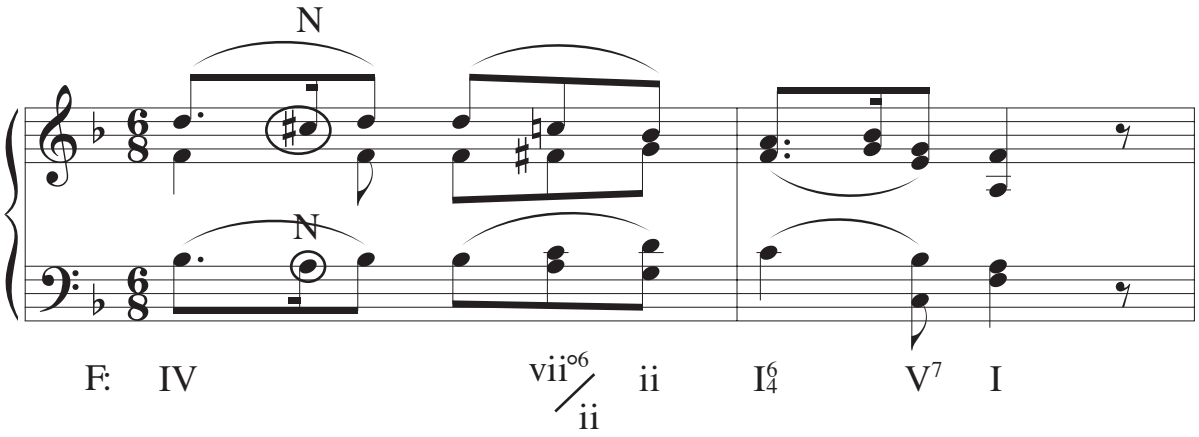


Secondary vii°?

You can also have secondary vii° triads (with no seventh), and even secondary vii^{ø7}'s, but these are much less common than the fully-diminished vii$^{°7}$ in Classical music.

Here is a snippet from a Haydn string quartet that sneaks in a plain vii$^{°6}$/ii as it switches between IV and ii.

Haydn, String Op. 20, No. 5 in F minor, third movement, mm. 7-8



Mode Mixture

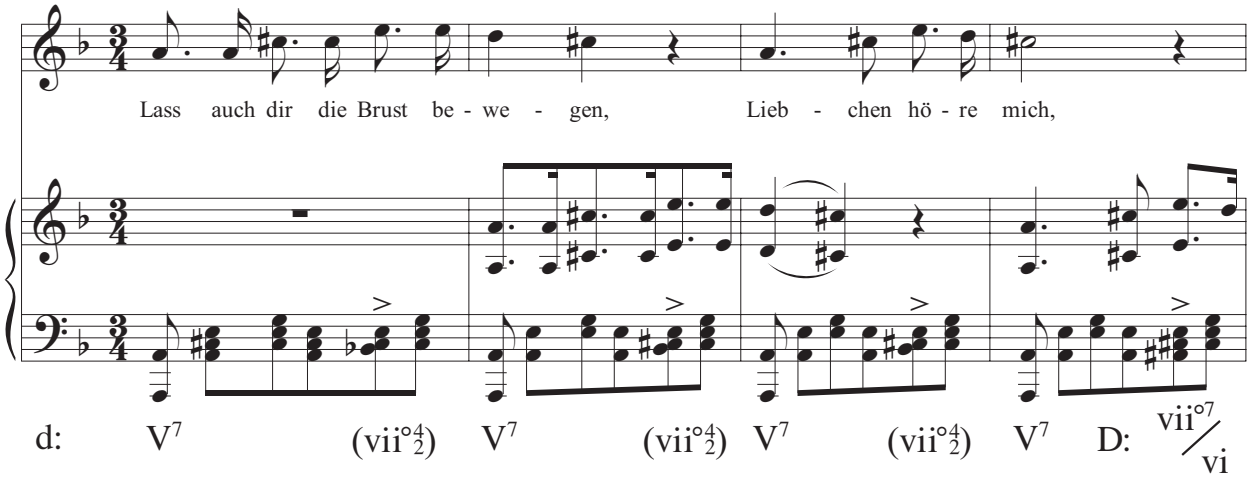
We will occasionally see chords that pop out of the key but don't seem to be functioning as secondary dominants. Another chromatic phenomenon you might observe is "mode mixture," the idea that the composer is "borrowing" harmonies from minor into major or vice-versa. Usually mode mixture injects minor-key chords into a major key.

When we encounter such a borrowed chord there really isn't any special notation to use - you can just apply our usual vocabulary of roman numerals to best describe the chord.

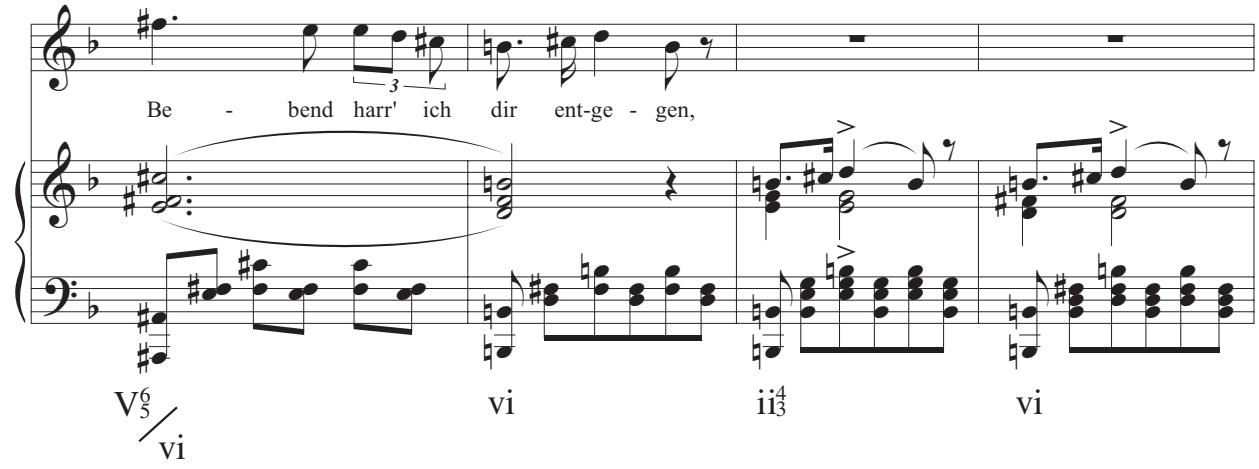
For instance, here is a Schubert song that starts in D minor and shifts to D major at the end. However, even after that happy transition he introduces more reverberations of minor.

Schubert, "Ständchen" (from the song cycle *Schwanengesang*), mm. 19-40

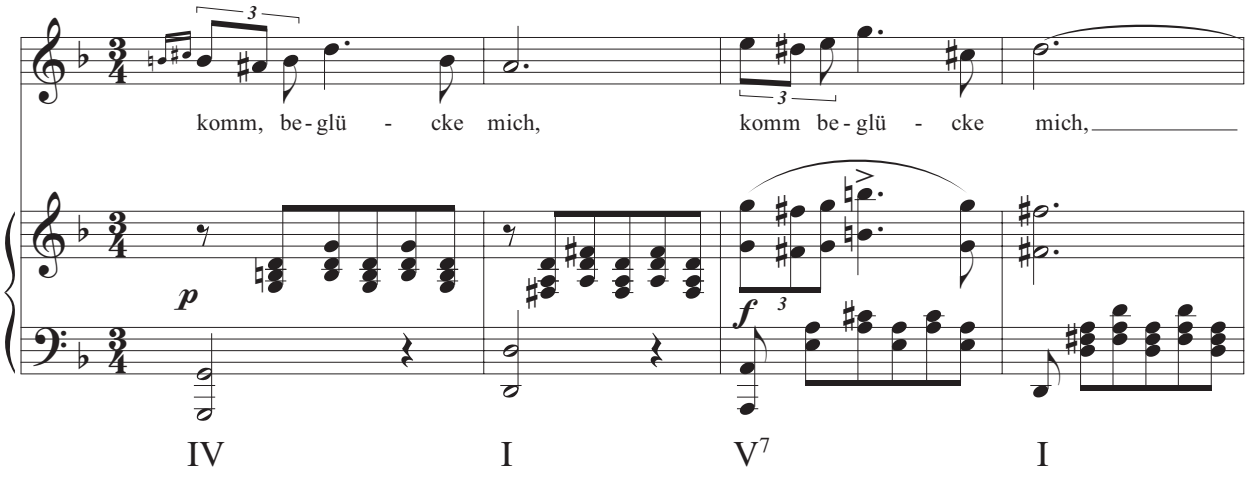
We'll start our excerpt at the most turbulent part of the song. The A7 could belong to either D minor or D major, but the B♭'s imply a little vii$^{\circ 7}$ and make it sound like we are definitely in minor.



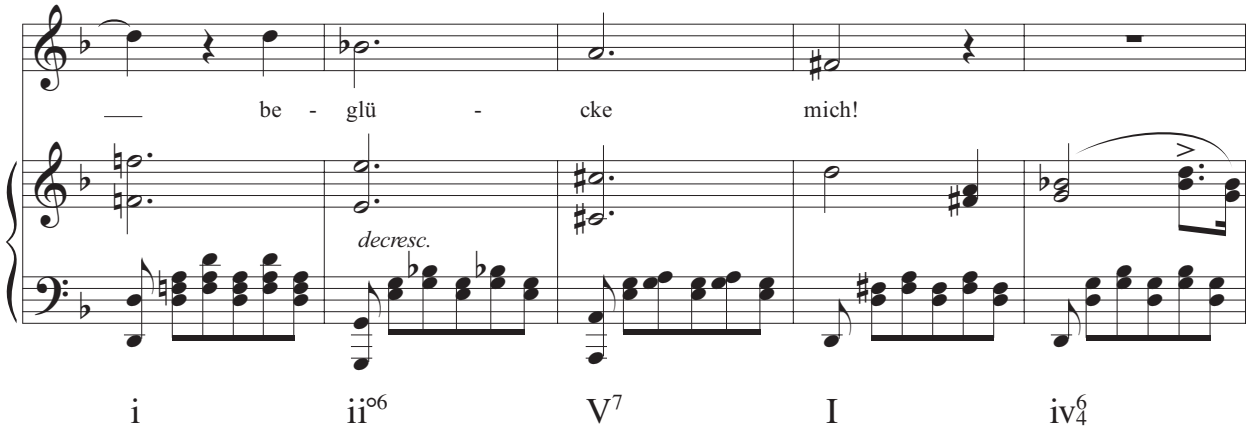
A secondary dominant pushes us up to vi of D major. This remote harmony still sounds dark and unsettled.

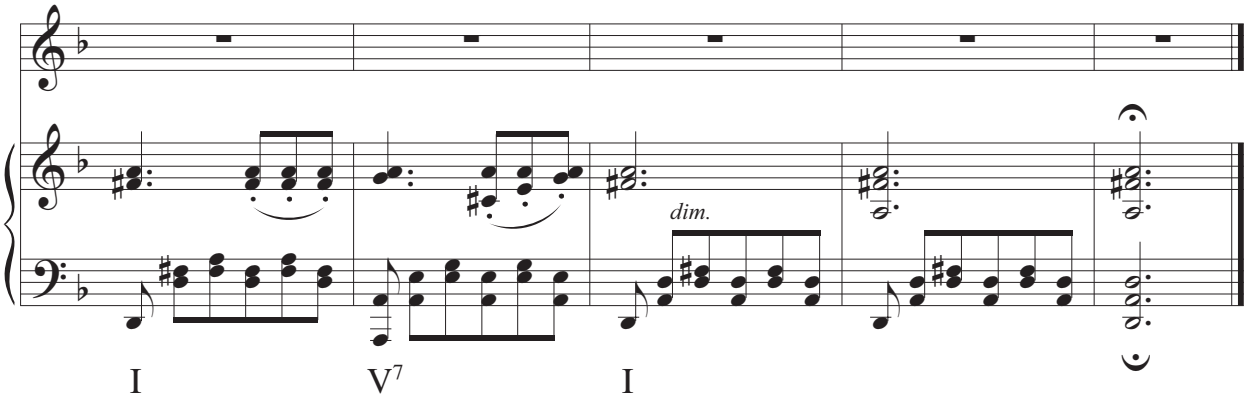


...but as we land on IV and I it is clear that this is major.



A few minor harmonies are mixed in.




dim.
I
V7
I

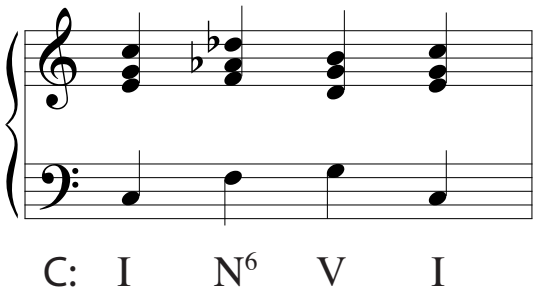
Chapter 7: More Unusual Classical Harmonies

There are a few more special chromatic chords in the typical harmony textbook which we aren't going to worry about in class. However, I will quickly summarize them here! If you are the sort of person who wants to know *everything* about music, this chapter is for you. (Also, if you ever find yourself working on a piece of Classical-type music and see something you don't know how to deal with, you can check back here and see if this chapter helps.)

The Neapolitan 6

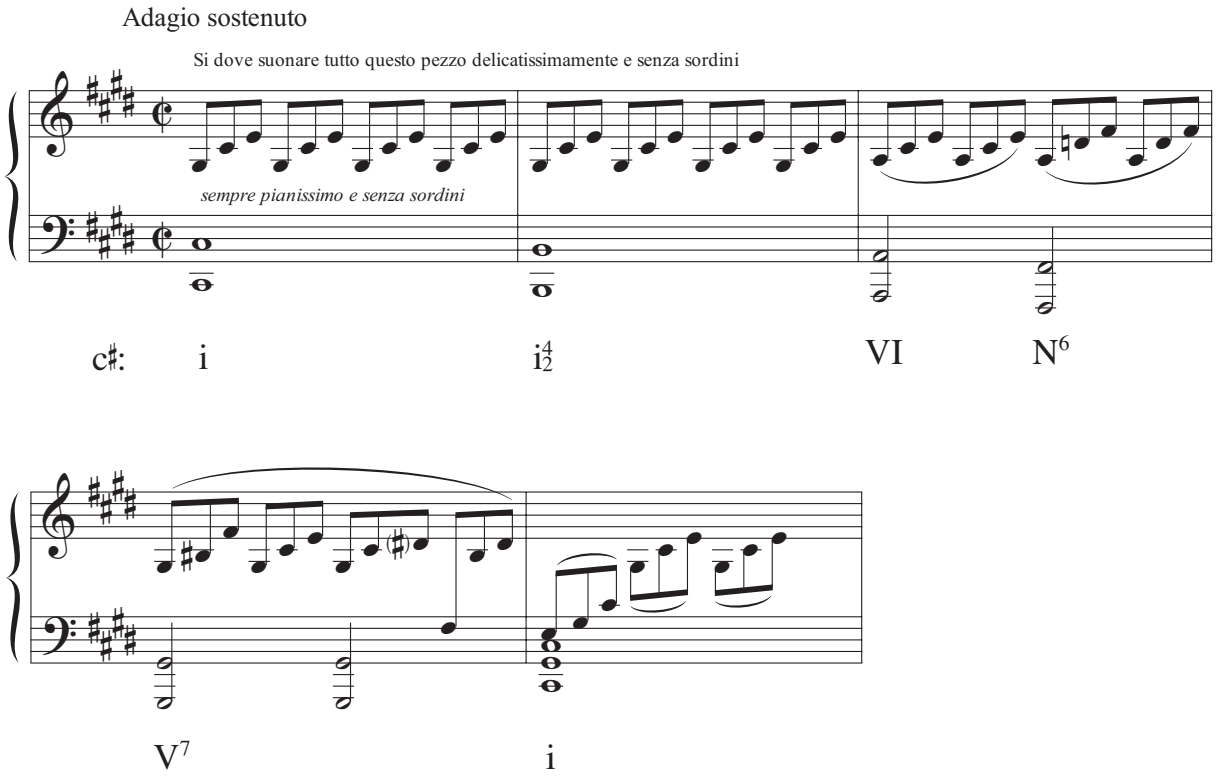
The Neapolitan chord is simply a major triad in first inversion that is based on the flattened scale-degree $\hat{2}$. We *could* label it as $\flat II^6$ instead of the more typical ii^6 or $ii^{\circ 6}$. Textbooks are enamoured of its special status, however, so they often go with N^6.

Here is a very basic example in C major:



... and the first moments of Beethoven's popular "Moonlight" sonata present a nice example.

Beethoven, Piano Sonata No. 14 in C-sharp minor, Op. 27 No. 2 "Moonlight", first movement mm. 1-5



The Augmented Sixth Chord

The augmented 6th harmony sounds a lot like a dominant 7th chord, but it is spelled in an unusual way that resolves "inside out", usually proceeding to V or I$^{6}_{4}$.

There are three variants which have regional names. Let's consider the simplest one first, the "Italian" aug 6th. As usual I will diagram one in C major.

The elements of an It+^{6} are...

F♯ → G

a ♯$\hat{4}$, a half-step below the dominant. This wants to resolve up, to $\hat{5}$.

C → D, B

one or two tonic notes, $\hat{1}$. These want to resolve up to the fifth of the V chord ($\hat{2}$) or down to the third ($\hat{7}$). If you have two of these in your four-note texture you can split the difference and go both ways, covering the V chord nicely.

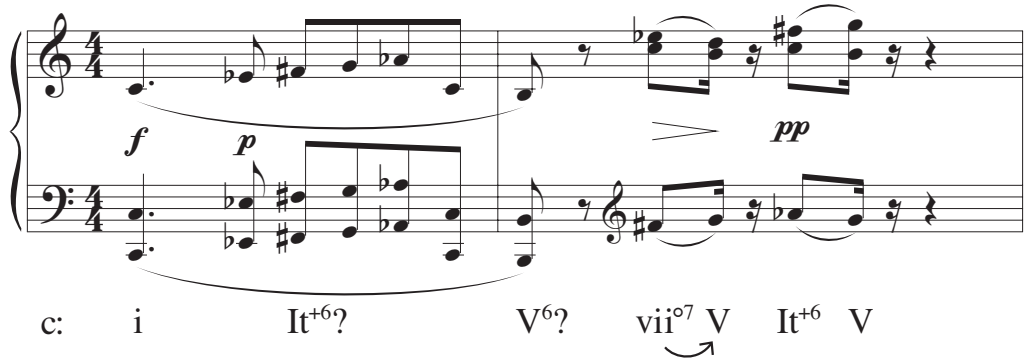
A♭ → G

a ♭$\hat{6}$. This also wants to slide down to the dominant.

C: It^{+6} V

The interval from ♭$\hat{6}$ to ♯$\hat{4}$ is an augmented sixth, a stretched-out span that is the same size as a minor 7th. That's why this harmony sounds like a dominant seventh but does not behave like one.

Mozart does one of these early on in his C Minor Fantasia, K. 475. Here's measures 1-2:



French Augmented Sixth

The French augmented sixth chord adds one new tone to the basic formula of the It^{+6}. Instead two $\hat{1}$'s, this version adds both $\hat{2}$ and $\hat{1}$ for a "whole-toney" sound that is really kind of exotic.

F♯ → G ♯$\hat{4}$ resolves up to $\hat{5}$.

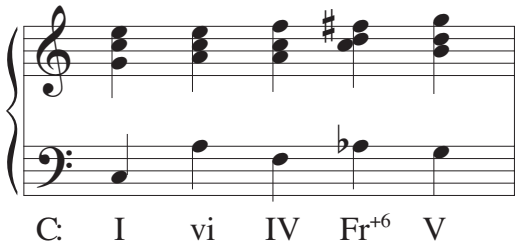
D → D The new $\hat{2}$ holds over as a common tone.

C → B Tonic goes down to B.

A♭ → G ♭$\hat{6}$ resolves down to $\hat{5}$.

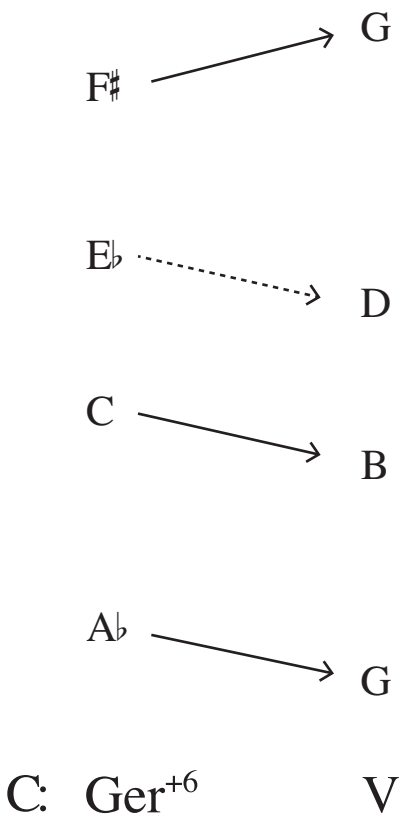
C: Fr^{+6} V

Here is a basic keyboard progression, to demonstrate:



German Augmented Sixth

The German version adds a minor $\hat{3}$ instead of a $\hat{2}$, which perfectly mimics the sound of a full dominant 7th chord on ♭VI.

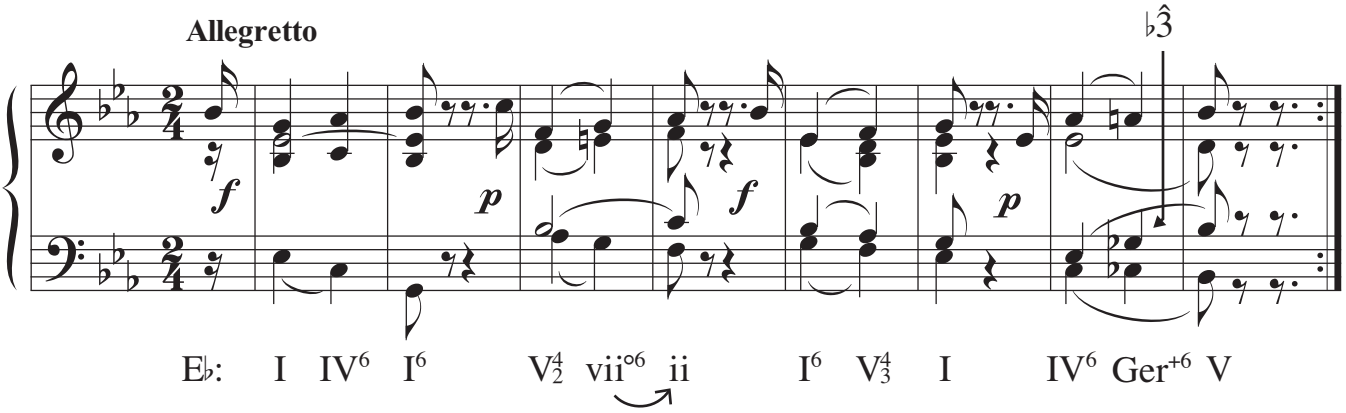


Our minor $\hat{3}$ very much "wants" to go to the fifth of the V chord, but doing so will create parallel fifths with the bass.

C: Ger^{+6} V

In this string quartet excerpt Haydn dodges the problem by flipping up his viola line up to the $\hat{5}$, tripling the root of V.

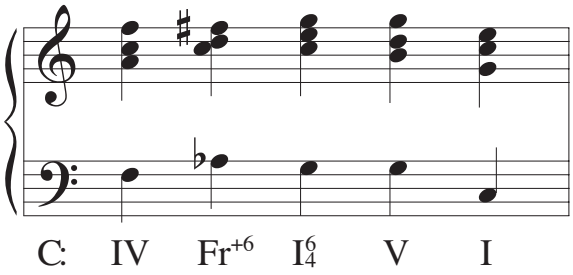
Haydn, String Quartet in E-flat major Op. 76 No. 6, first movement mm. 1-8



Augmented Sixth Chord to Cadential 6-4

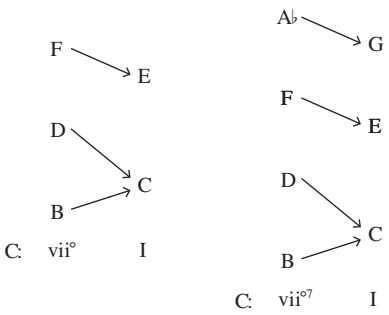
I've diagrammed all of our augmented sixth chords as though they always go directly to V. However, another common move is to resolve to an intervening cadential $\frac{6}{4}$.

Here is a basic keyboard harmony example:

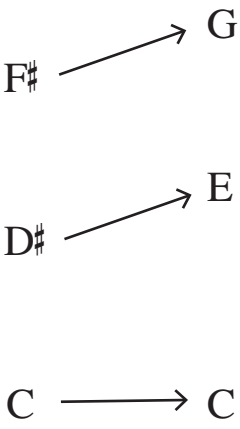


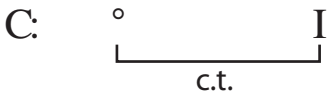
Common-Tone Diminished Chords

So far we've seen lots of diminished triads and seventh chords that resolve inwards, like so:



Occasionally you'll see diminished harmonies that are instead treated like "deflated" major chords. The composer leaves one note on the bottom and reinflates the rest of the triad so that the upper tones slide up.

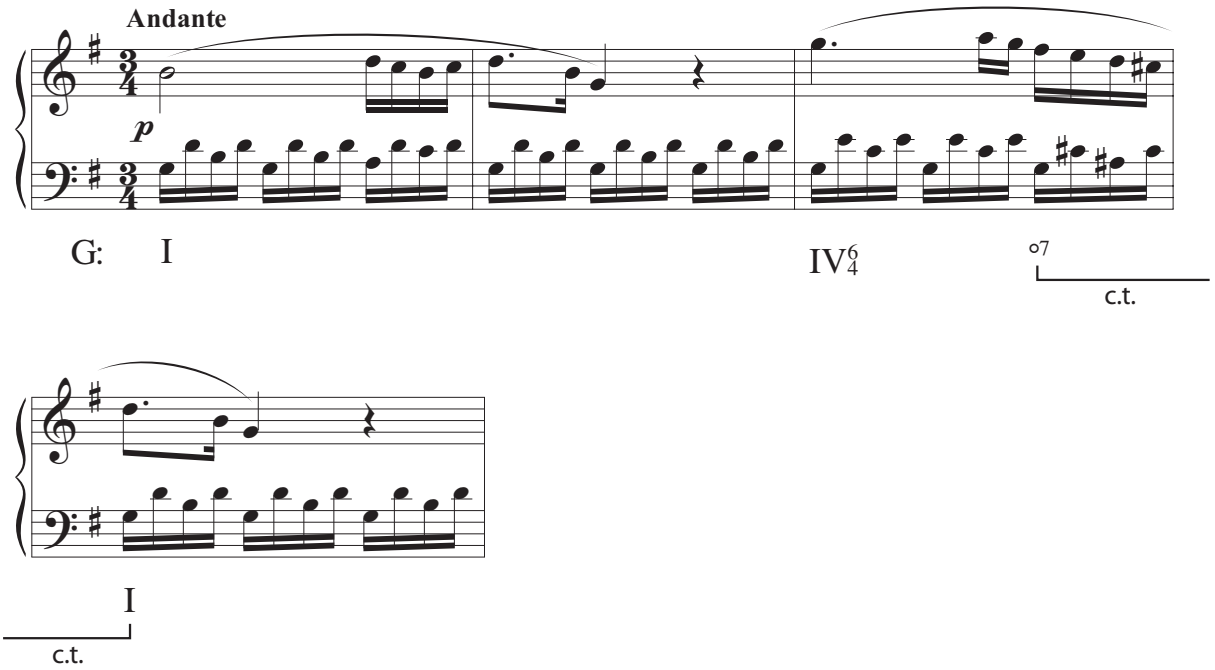




The way I learned to mark these is to make the diminished circle, a bracket leading to the target chord, and a "c.t." for common-tone.

Common-tone diminished triads (and sevenths) are frustrating to analyze because they are spelled like a random diminished chord that doesn't resolve where you would expect. In the example above, for example, it looks like a D♯ diminished seventh which would be vii°$^4_2$ of E. However, as you look closer you see that it leads to a C chord, and realize that this is something different.

Here is Mozart doing it in the second movement of his Piano Sonata in C Major K. 545.



Chapter 8: Phrases and Cadences

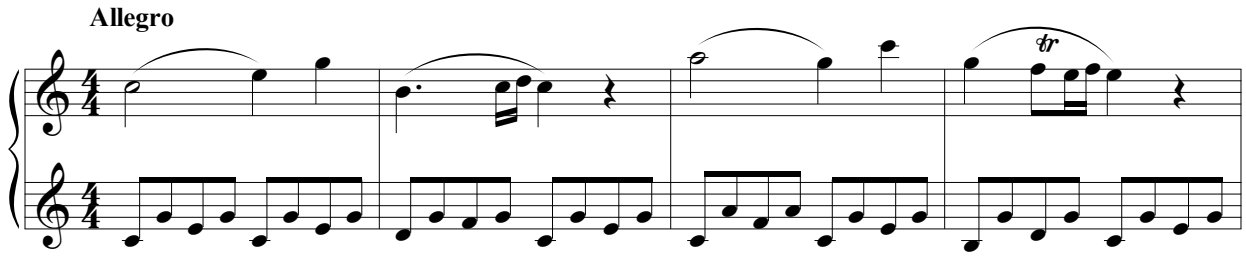
One of the final elements we need before we can really dig in to Classical music is an understanding of phrases and cadences. Taking stock of the way a piece organizes its ideas is an easy and pleasant way to familiarize ourselves with a piece and get the "big picture" of what's going on.

Phrases

Phrases are just short passages of music with a clear beginning and end. Sometimes these are very obvious, as we hear an idea that starts at one moment, flows onward for a little bit, and then pauses, creating a nice punctuating gap that we can hear with our ears and see in the sheet music.

For instance, this Mozart piano sonata begins with two short phrases. The left-hand part keeps moving, but the melody has clear pauses that we can see and hear.

Mozart, Piano Sonata No. 16 in C major, K. 545, first movement mm. 1-4



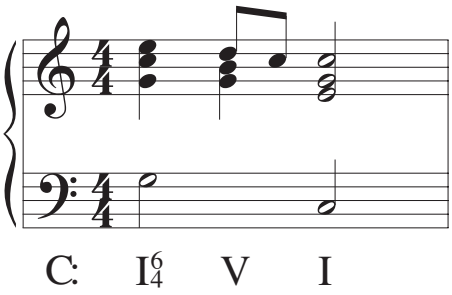
I like to diagram phrases with a version of the musical slur shape - the curved ends of the line represent the boundaries of each musical unit and we can indicate the length of the phrase by putting the number of measures underneath.

Cadences

Cadences are little harmonic and melodic formulas that punctuate the ends of phrases. In Classical music they are very standardized, and we have technical terms to define the different kinds we will see.

Perfect Authentic Cadence (PAC)

The PAC is the ultimate conclusive cadence. It's like the period at the end of a sentence. It will typically go V-I or V^7-I, and maybe it will have a cadential $\frac{6}{4}$ in there as well. Here is a made-up PAC in our typical four-voice texture.



(I added a little anticipation in there to make it more cadence-like.)

There are certain elements that are required for it to be a perfect authentic cadence.

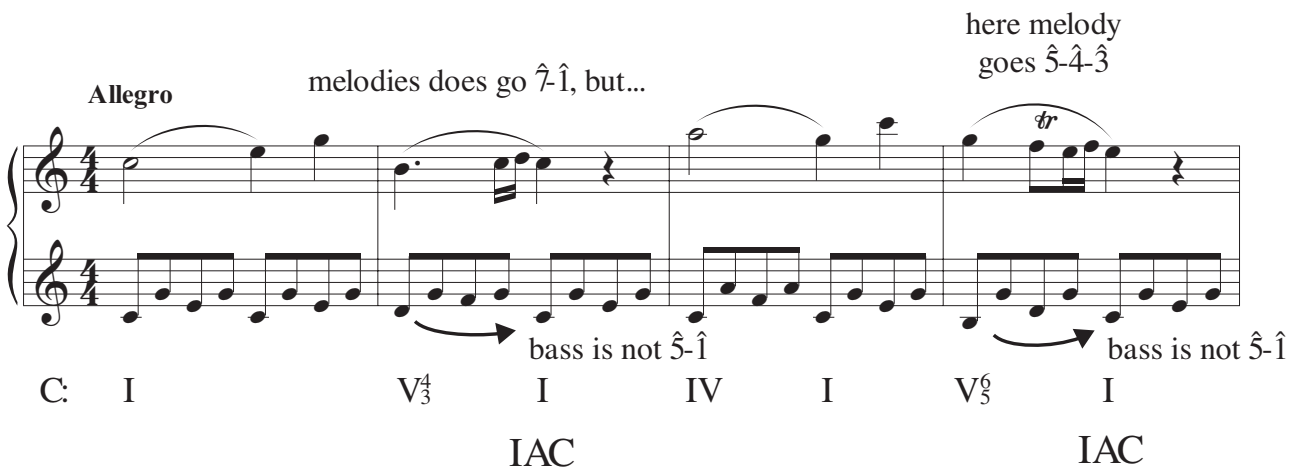
- The bass line must go $\hat{5}$-$\hat{1}$, making our harmonies in root position. Neither V nor I can be inverted.

- The melodic line must land on $\hat{1}$. It really should have a clear connection from $\hat{7}$ to $\hat{1}$ or $\hat{2}$ to $\hat{1}$.

Imperfect Authentic Cadence (IAC)

If either of those two "rules" are undercut, it's an imperfect authentic cadence. An IAC just isn't as conclusive and emphatic as a PAC.

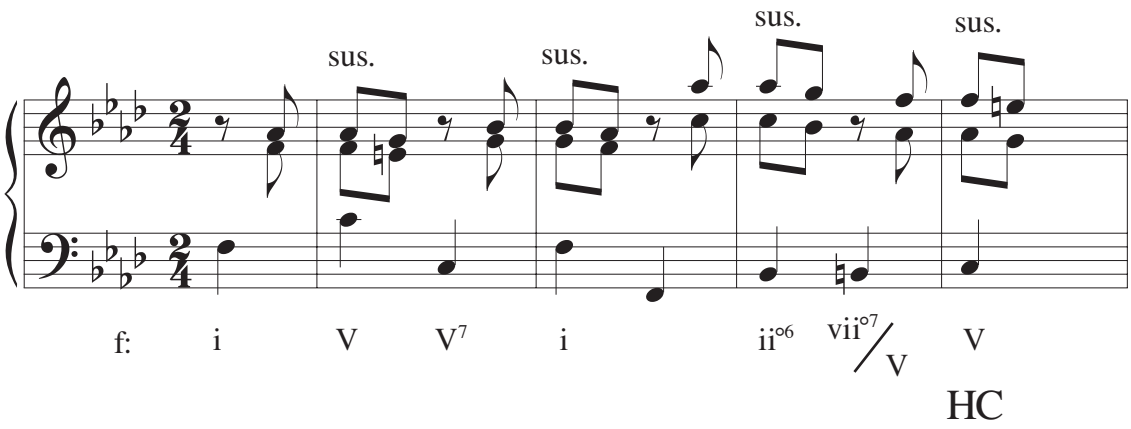
Our Mozart passage begins with two IACs. This makes sense because we are just getting started and our short phrases don't need strong endings.



Half Cadence (HC)

Half cadences simply pause on V. This creates a sense that the music is "up in the air" and still needs to land, demanding more music to finish the thought. It is like a comma instead of a period.

J. S. Bach, Prelude in F minor from The Well-Tempered Clavier, Book II



Plagal Cadence (Plag.)

Plagal cadences go IV-I. Like the IAC it is somewhat conclusive but not as emphatic as the PAC.

Deceptive Cadence (DC)

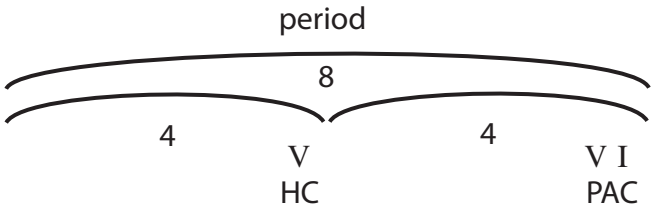
The deceptive cadence sets up a V-I motion but instead goes V-vi! As you can imagine, this is very inconclusive and demands more music.

Anything Else

If a phrase ends with anything else you might consider the possibility that there just isn't a cadence there, and the musical thought you are looking at is more like a "sub-phrase." However, you could argue that a pause on anything other than I is some kind of half cadence, and any unusual motion to I (like ii-I) is some kind of IAC.

Periods

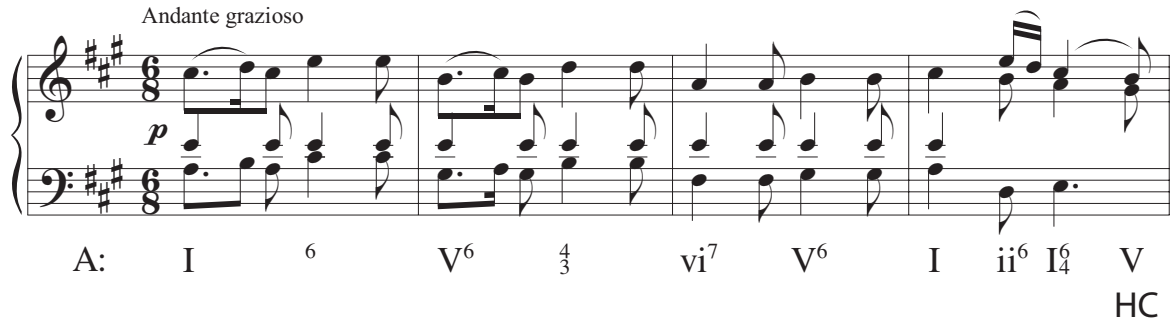
Composers will often write a phrase with a "weaker" cadence followed by one with a stronger cadence, which causes the two passages to hook up and form a larger unit. This is called a *period*. Perhaps the most common design for a period is a half cadence followed by a PAC.



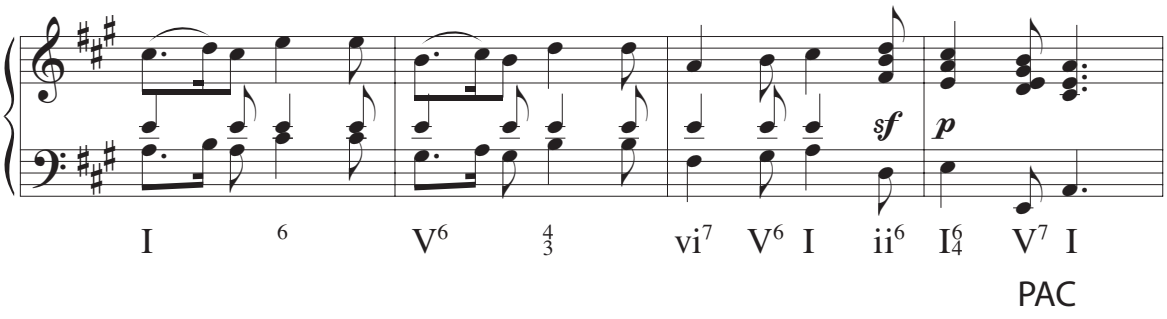
A weak cadence followed by a stronger one is sometimes called a "question-and-answer" relationship. In more formal academic writing the first phrase is the *antecedent* and the following phrase is the *consequent*.

A Parallel Period in Mozart

The first movement from Mozart's Piano Sonata in A Major, K. 331 (nicknamed "Alla turca") presents a good example of this kind of phrasing. The first four measures pause on a half cadence:

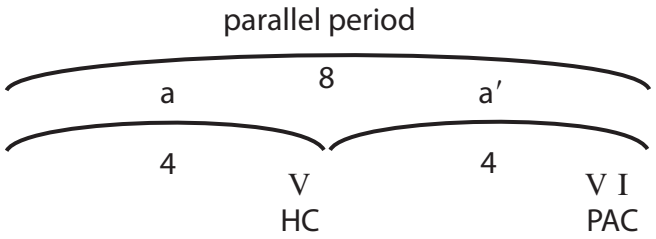


...and then the second four (mm. 5-8) conclude on a PAC.



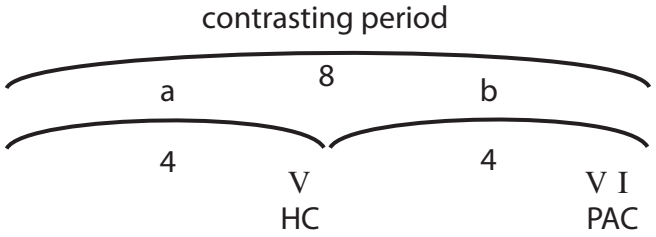
The Mozart K. 331 example from the previous page has two phrases that repeat the same idea, with the second part altered at the end to make a more conclusive cadence. This is a *parallel period*.

We can label the two phrases with a little letter a, to note that the material is the same. The second phrase gets an a′ (or "a prime") to indicate that it has been modified with a new ending.

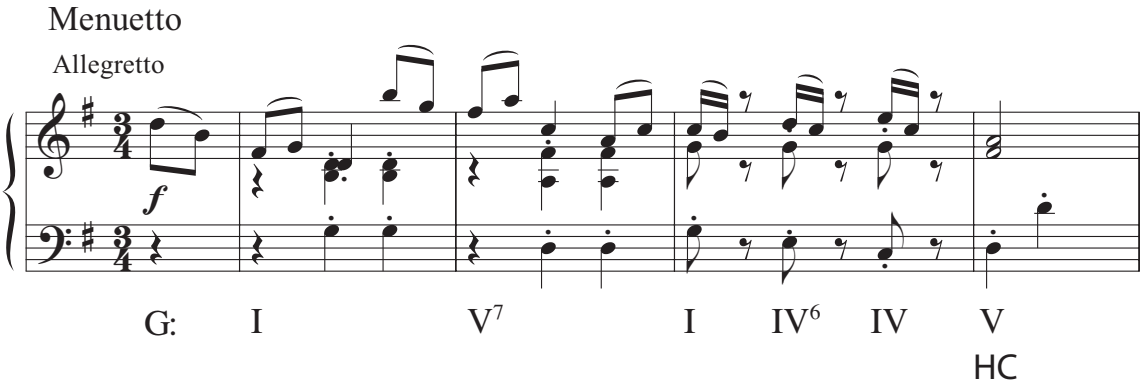


A Contrasting Period in Haydn

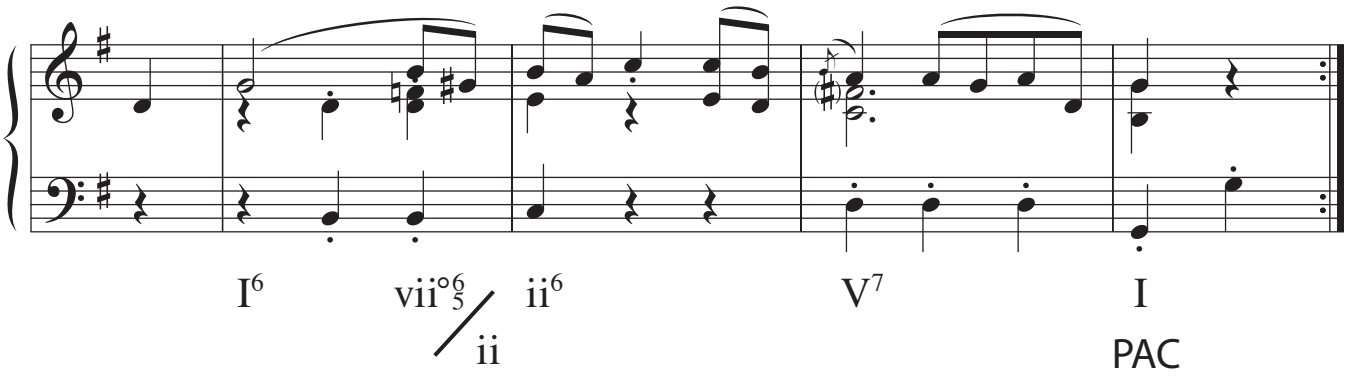
The opposite possibility is a *contrasting period*, in which our phrase with the weak cadence is simply followed by something new with a strong cadence. We could label our phrases a and b.



The second movement minuet from Haydn's String Quartet in G major Op. 64 No. 5 presents a nice contrasting period right off the bat. The first four measures present short, punchy figures that lead up to a half cadence:



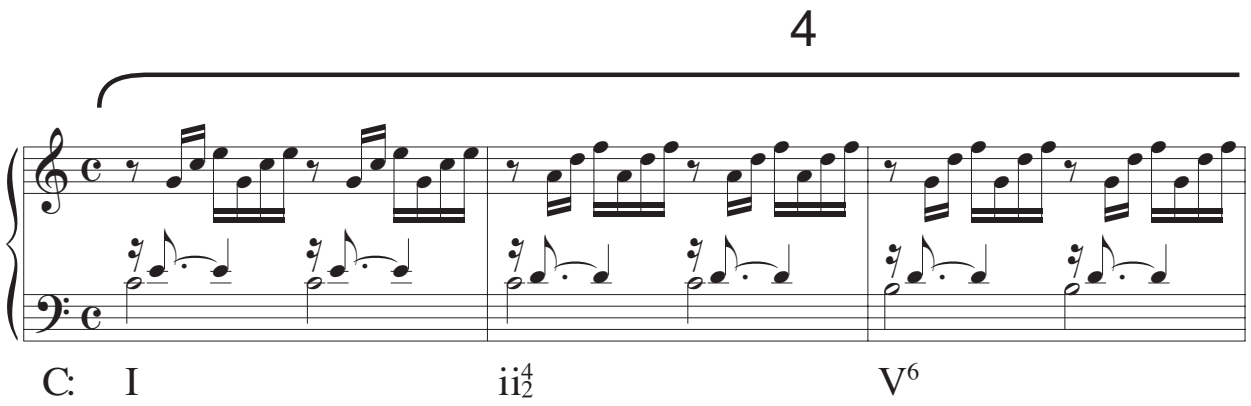
The next four measures answer with longer, more *legato* figures that lead into a PAC.



Phrases Without Pauses

It's possible to hear phrases in music that doesn't actually pause, rolling out ideas in continuous "blocks" or "chunks" that flow into one another. Let's look at the J. S. Bach's Prelude in C major from *The Well-Tempered Clavier Book I* as an example.

Perhaps when we first look at the score we see no visible evidence of phrasing - it just looks like a constant stream of arpeggios. However, when listening it is easy to hear how the first four measures come to a close, making an IAC on C major. Then, the next block travels to G major and concludes with a slightly stronger IAC on G. Our sense of harmony and cadences can shape our experience of this piece and help us parse it into phrases.



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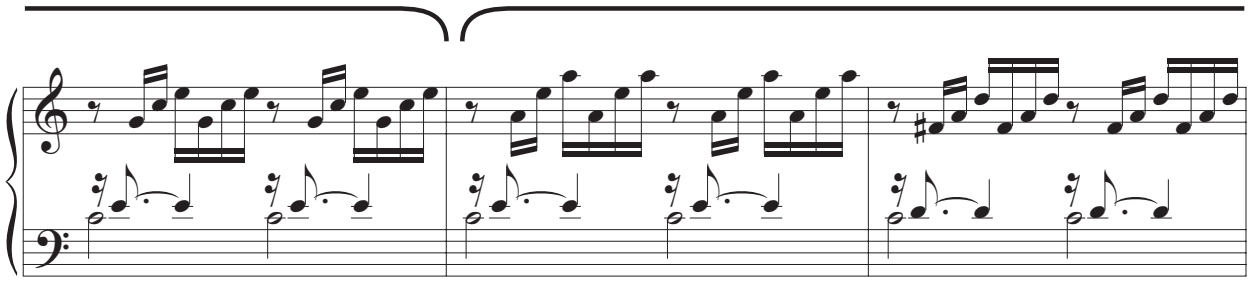
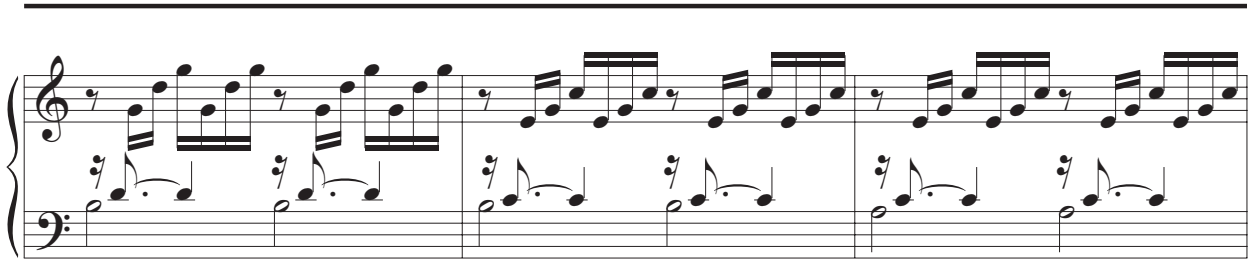
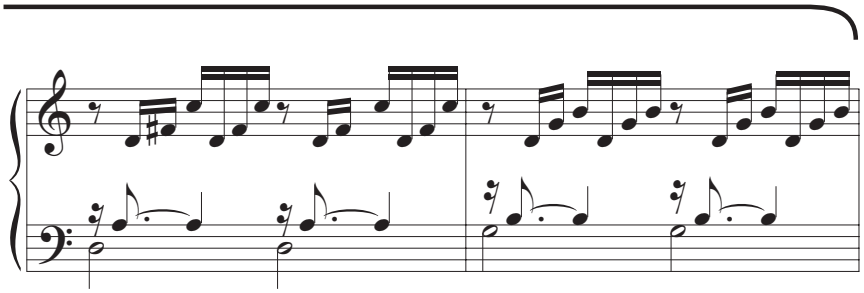


I
IAC

modulating to G major....





G: V^7

I
IAC

Chapter 9: Modulation

Modulation refers to the practice of changing keys after a piece has begun. This phenomenon is central to most Classical-type music, because composers apparently felt that moving from one key to another makes music more interesting. So a typical Classical piece starts in one place (the original key), travels to at least one new key, and eventually returns back to where it started. In certain fairly large-scale forms (like a sonata-form movement) the interplay of keys creates a sort of story or argument.

How do we know when a piece is modulating?

- Sometimes pieces intentionally create the perception that we are "picking up and moving to somewhere else." The musical language may sound very dynamic and unstable.

- In skimming the score (perhaps while listening to a recording) we may notice an arrival on a cadence in a key other than the tonic.

- The underlying scale of the piece may change noticeably, as the composer consistently introduces new flats or sharps that spell a new scale that does not correspond to the key signature.

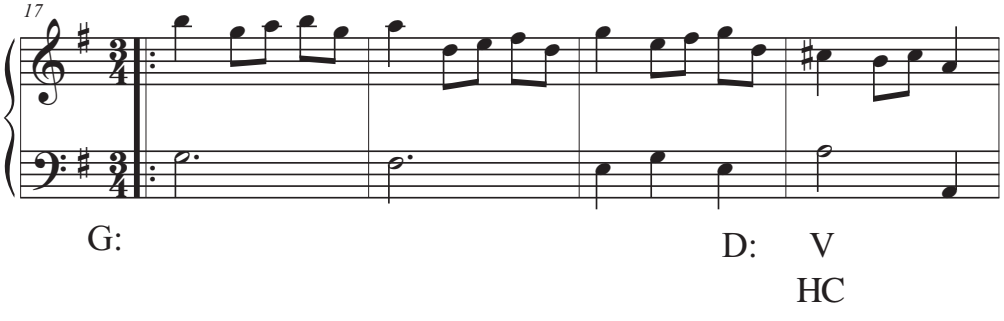
- As we look at the harmonies and think about a roman numeral analysis, we may notice chord progressions that do not seem consistent with our usual flowchart. Perhaps they make more sense in a new key.

Dealing with modulation in a large-scale view

If you are taking a birds-eye view of your piece (i.e. looking for big ideas and not labeling every chord), you might want to analyze modulation by simply placing a new key indicator roughly where you think the tonality changes and labeling whatever you need to label (like cadences) after that.

For instance, for homework we sometimes look at J. S. Bach's Minuet in G (from the *Notebook for Anna Magdalena Bach*.) It starts in G major, but as we get into the second half of the piece we see a cadence that looks like a big V in D major. We could drop the D: key indicator and mark our half cadence at that point, and maybe that's all we need.

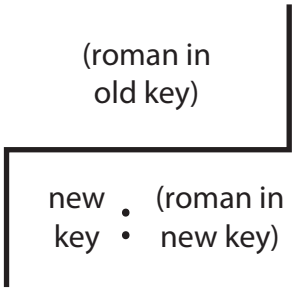
Bach Minuet in G with only the key change and cadence labeled



The Pivot Chord

However, if you are really trying to understand the note-by-note fabric of a piece, you want to know about a traditional analytic concept called the *pivot chord*.

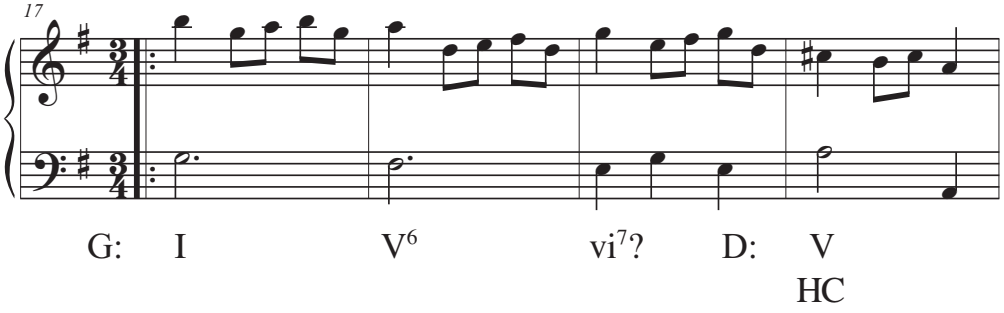
The idea of the pivot chord is that it has a valid flowchart function in both the old key and the new key. To mark your pivot, you want to find the last roman numeral that makes sense in the old key. Draw a sort of zig-zag shape that terminates the old key and begins the new one like so:



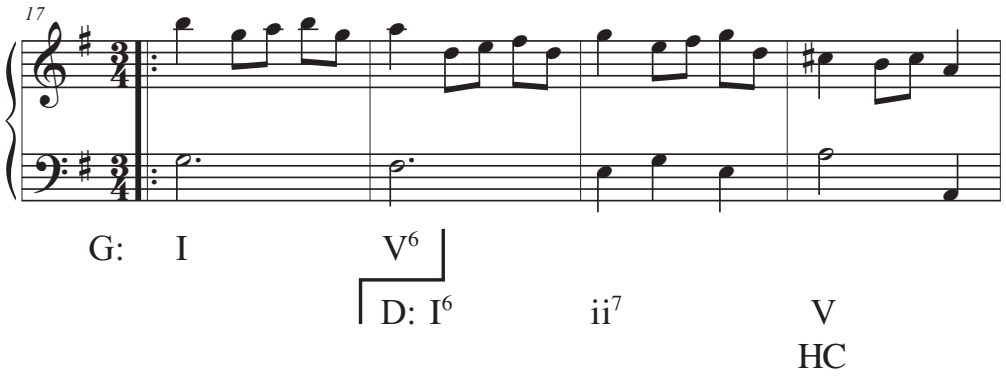
Let's try to do a more detailed analysis of our Bach Minuet modulation, complete with pivot. First, we'll go as far as we can in G major before we are forced to understand things in D major.

Measure 17 is I in G, so that's pretty simple. Measure 18 moves to V^6, which also makes sense in G, and then measure 19 goes on to vi^7, which is maybe unusual but certainly not unheard of. But by measure 20 we are definitely in D major, with a big V that simply doesn't belong to G.

Analyzing the Minuet modulation mostly in G major



I'm going to be very conservative and decide that the vi^7 "doesn't make sense" in G. I'll make a pivot in m. 18 instead, with chord that makes perfect sense in both keys. (See next page.)



So we've decided that the harmony in m. 18 is V^6 in G *and* I^6 in D. Then the chords that follow all make sense in D, with a very textbooky motion from I^6 to ii^7 to V.

If you wanted to put your pivot a measure later and analyze the e chord as both vi^7 and ii^7 I suppose that would also be fine.

The pivot doesn't sound that special

The most counterintuitive thing about the pivot chord is that it probably doesn't sound special - it doesn't "pop out" of the music and signal that something new is happening. Instead, it is the *invisible seam* that connects the two keys as one flows into the other. It is really the next chord (the first one that is definitely in the new key) that is likely to pop out and sound different, but the pivot ensures that the next chord doesn't sound random or arbitrary.

Classical composers had limited pathways between keys

Because Classical composers usually wanted a "smooth" and easy-to-understand transition between tonal areas, they usually limited their modulatory paths to related keys that correspond to chords in the original key. For a major key you can make a little table that includes I, IV and V, as well as the "relative minors" of those (vi, ii and iii.) These are the closely-related keys that would be easy to travel to with a nice pivot chord. (Another way to think of this is that these scales are no more than one flat or one sharp away from the home key on the circle of fifths.)

The closely-related keys for C major

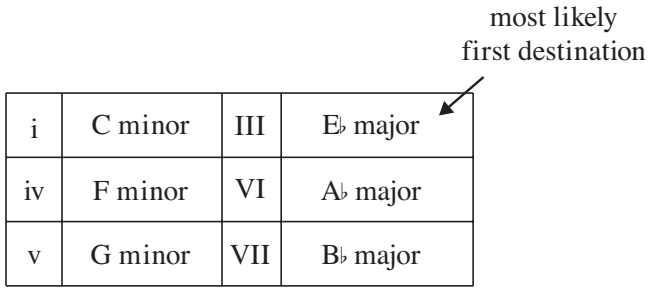
I	C major	vi	A minor
IV	F major	ii	D minor
V	G major	iii	E minor

most likely first destination

In major keys the most likely destination is the key of the dominant. Sometimes a piece with travel to the V key first and then to more exotic ones later.

Minor keys work basically the same way. We can consider the i, iv and (minor!) v along with their relative majors. Here the most common first destination is the relative major of tonic (i.e. III).

The closely-related keys for C minor



i	C minor	III	E♭ major
iv	F minor	VI	A♭ major
v	G minor	VII	B♭ major

Sometimes there is no pivot chord

Sometimes a composer will change keys using some other technique that doesn't involve a smooth pivot. In these situations you can simply drop a new key indicator into your analysis and start analyzing chords with the new set of romans.

Chapter 10: Rock Harmonies

It may not surprise you to learn that the ideas we've developed about chord progressions in Classical-type music have some relevance for other styles, but they are not a perfect fit! Certain practices that are very common with composers such as Bach, Mozart and Beethoven are rarer in more contemporary genres, and popular music has a tendency to do things that a Classical composer would not do. This chapter aims to expand our vocabulary so that we can describe progressions that you might see in rock music.

In general I'm using the term "rock" as a stand-in for many different kinds of music. These ideas here might also apply to chord progressions in pop, jazz, country, R & B, hip-hop, all sorts of styles. (One detail that is fairly rockish, however, is the focus on mostly plain triads, while "jazzier" styles have more seventh chords and other complexities.) I'm going to stick to examples where there is an instrument playing simple chords, since analyzing harmonies will get more complicated if a track features complicated riffs or other figurations.

I. Rock tunes that stick to our flowchart

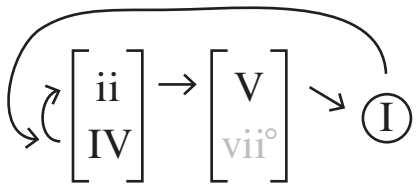
It is not impossible to find popular songs that stick closely to the traditional progressions we've already studied. One example I've cited in class is "12:51" by the Strokes.

```
E                   A
Talk to me now I'm older

     F♯m                          B
Your friend told you 'cause I told her

E                         A
Friday nights have been lonely

F♯m                             B
Change your plans and then phone me
```

Rock tunes usually don't pause on cadences like Classical music does - instead the music is usually more continuous and fluid. Here there is a four-measure cycle that repeats the same four chords over and over again. It sounds like we land on the "home chord" (E major) at the beginning of each cycle. We could analyze all four chords in E major and get a fairly familiar-looking progression of roman numerals:

E: I IV ii V

Obviously this progression starts on the tonic, goes out to a subdominant harmony (IV), flips to a different subdominant (ii), and then proceeds forward to dominant and back to tonic on the next loop.



A deceptive progression

We even see some of our "flowchart exceptions" from Chapter 4 in pop music. You will sometimes hear a tune go from V up to vi in a "deceptive" move that seems to extend the musical momentum.

For instance, "She's Got a Problem" by Fountains of Wayne goes out to vi as it gets towards the end of the verse, in order to set up a final winding path back to I. We are in the key of F major, and I'll quote it starting at 0:49 as it vacillates between IV and V.

```
         B♭              C
She's a danger to herself

         B♭                   C
And I'm worried about her health

              Dm       Am          B♭           C          F
She's got a problem, and she's going to do something dumb.
```

After four bars of lingering on IV and V, it kicks out to vi (D minor), then iii, IV, V and I. The use of iii on the way to IV is a little unconventional, but it does happen occasionally in Classical music as well.

F: IV V IV V vi iii IV V I

deceptive progression

II. Secondary dominants

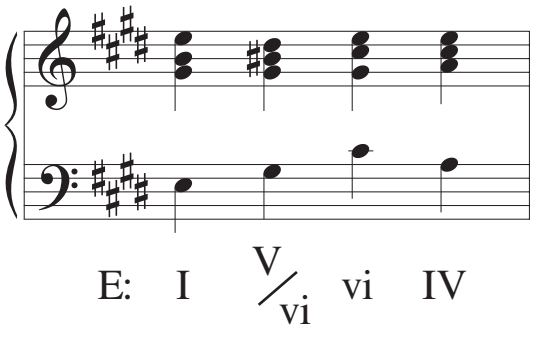
Rock music also will use certain familiar chromatic chords to push the progression in new directions.

"Sink to the Bottom" by Fountains of Wayne is a very straightforward example of a secondary dominant. Like in our Strokes tune this is another repeating sequence of four chords, in this case E, G♯, C♯ minor, and A.

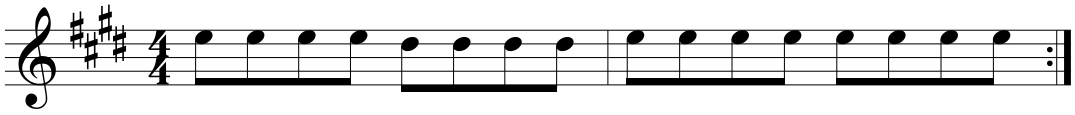
```
          E             G♯                C♯m       A
I wanna sink to the bottom with you,

E            G♯                C♯m       A
Sink to the bottom with you,

E             G♯              C♯m       A
The ocean is big and blue,           I just wanna

E            G♯                C♯m       A
Sink to the bottom with you.
```

Once again we are in E major. If we just put roman numerals on the chords we could call them I, major III, vi and IV. That III is acting as a secondary dominant to vi (like a V-i in the key of C-sharp minor), so we might as well put that in our analysis.

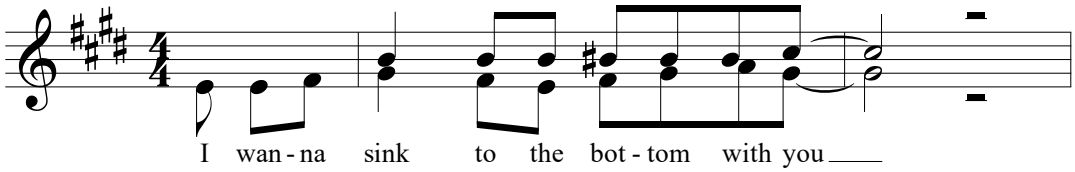
Writing out the progression in our typical keyboard style also shows a few voice-leading paths that show up very explicity in the song.



Our soprano line goes do-ti-do-do, which we hear in the song as a background figure on synth.

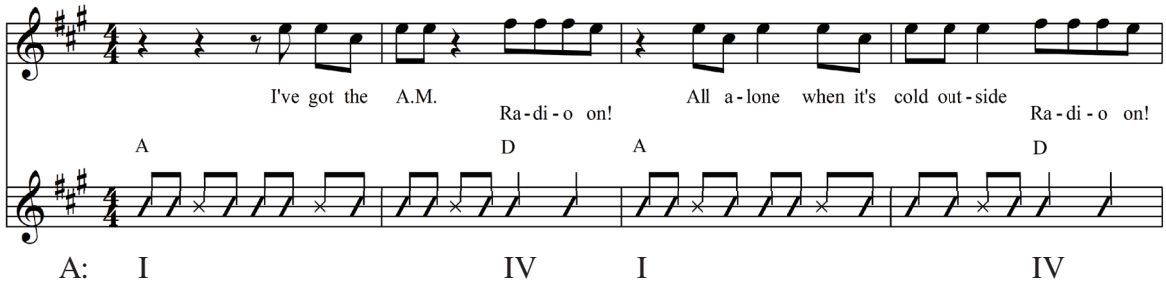


...and the B-B♯-C♯ line in the alto occasionally appears as a harmonizing vocal part.



III. The plagal progression and "plagal stacks"

Popular music seems to like the plagal progression a lot more than Classical music does. My personal theory is that using the IV sounds fresher and less "square" than the more traditional V. In an earlier chapter I cited The Modern Lovers' "Roadrunner" as a tune that relies heavily on the I-IV-I motion.



(The Modern Lovers tune does reserve V-I for the final cadence, however, perhaps recognizing that it is still "stronger" and more conclusive than IV-I.)

Rock music will also happily go V-IV-I, a progression that almost never happens in Classical music. This goes back to the Blues roots of Rock 'N Roll, as V-IV-I is a common final cadence in a traditional 12-bar Blues song. For example, let's consider "Sweet Home Chicago." (I am using Magic Sam's 1967 recording as a reference)

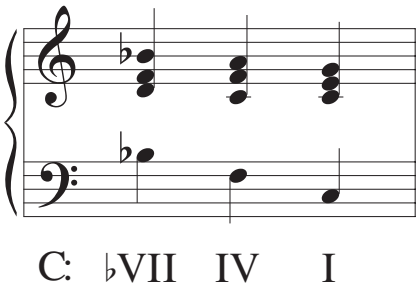
```
E7                  A7                          E7
Come on!            Baby don't you want to go.

A7                                              E7
Come on!            Baby don't you want to go.          Back to that

B7                  A7                          E7              (B7)
same old place,               sweet home Chicago.
```

Here our progression is in E, and we see the V^7 IV^7 I^7 cadence on the last line.

Plagal stacks

One sometimes hears a sequence of ♭VII, IV and I in popular music. I like to call this a "plagal stack," as the motion from ♭VII to IV is down a fourth, just like IV to I. We could even argue that ♭VII is functioning as "IV of IV," though this concept of a secondary plagal is perhaps not as versatile as a secondary dominant.

Here is a basic plagal stack written out in C major:

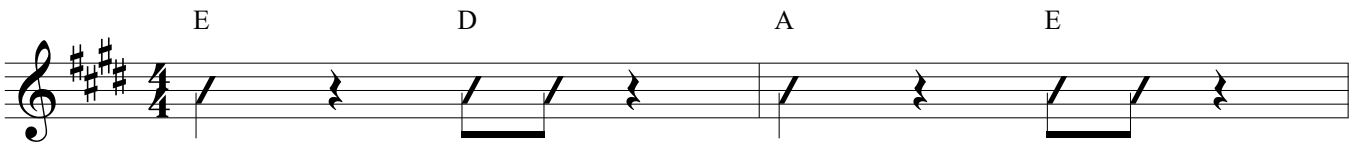


Let's look at an example in "With a Little Help from My Friends" by The Beatles. Here's the first verse:

```
E               B/D♯        F♯m
What would you think if I sang out of tune?

          F♯m           B7          E
Would you stand up and walk out on me?

E             B              F♯m
Lend me your ears and I'll sing you a song

         F♯m         B7          E
And I'll try not to sing out of key


          D                 A            E
Oh, I get by with a little help from my friends

          D                   A            E
Mm, I get high with a little help from my friends

          A                                E
Mm, gonna try with a little help from my friends
```

Once again we are in E major. The first stanza has chords that backpedal from I to V to ii, then proceed forwards again as ii-V-I. When we get to the hook of the song, we see the plagal stack - ♭VII (D major), IV (A major) and I.

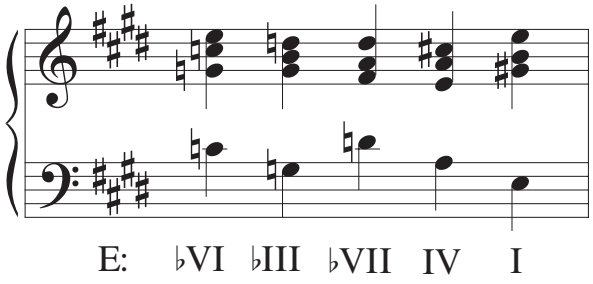
Another iconic example from a British Invasion band would be main riff from The Who's "I Can't Explain" [1965]. Also in E:



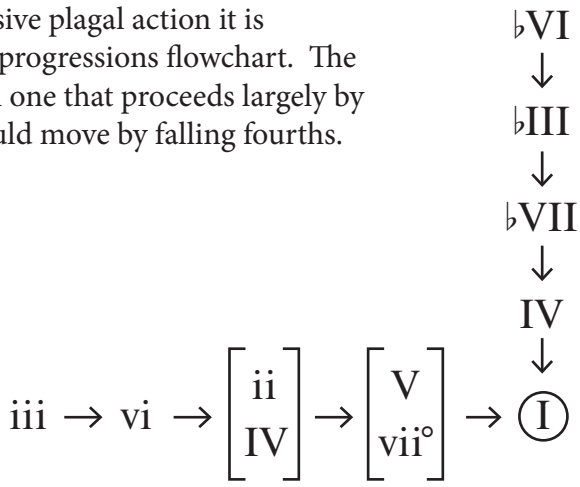
Perhaps the most spectacular plagal stack in all of rock n' roll occurs in Jimi Hendrix's "Hey Joe." We are driving towards E major, but we pass through a series of chords that don't seem to belong to the key.

```
C               G
     Hey Joe...

D               A
                    Where you going with that

E
gun in your hand?
```

This is a series of four plagal chords in a row, a chain that runs from ♭VI all the way to tonic.



In considering progressions with extensive plagal action it is tempting to forge a second axis for our progressions flowchart. The horizontal axis might be our traditional one that proceeds largely by falling fifths, and a new vertical axis could move by falling fourths.



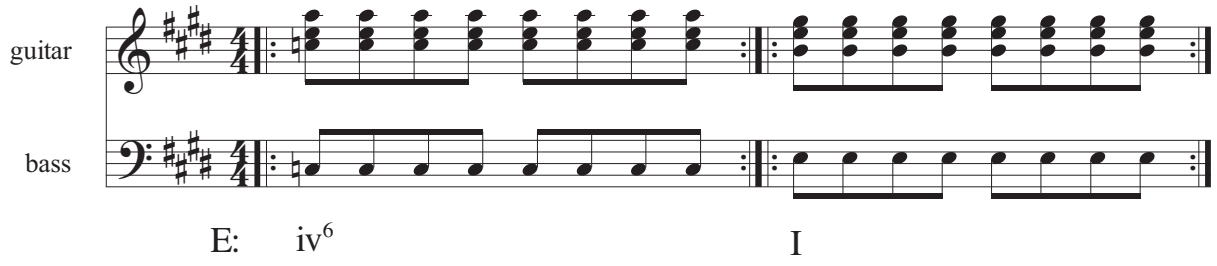
IV. Mode mixture

You may recall that mode mixture simply involves flipping between major and minor, usually injecting minor-key chords into a passage that is predominantly major. We saw this at the beginning of Chapter 6, when we considered David Bowie's "Space Oddity."

```
C                                   E
This is Ground Control to Major Tom

                          F
You've really made the grade

         Fm               C                            F
And the papers want to know whose shirts you wear

          Fm                   C                        F
Now it's time to leave the capsule if you dare...
```

We are in C major, and the switch from IV (F major) to iv (F minor) is a nice instance of modal mixture.

The intro to The Strokes' "12:51" also justaposes a major I with a minor iv. The tune is in E, and we move to a iv in first inversion, resolving to back to I.



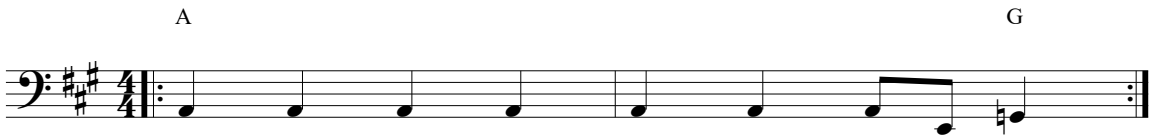
The song returns to this progression in the bridge as well, providing a little tonal contrast to the otherwise diatonic harmonies of the rest of the song.

V. Neighbor chords

We learned about neighbor tones and passing tones in our Chapter 3, when we were studying our non-chord-tones. These are kinds of melodic decoration, and a musician will use these to either travel from one stable place to another (in the case of a passing tone) or to decorate a more fundamental note (with a neighbor).

These methods of elaboration are not strictly confined to non-chord-tones, however. There are often cases where there seems to be a continuity between NCTs and harmonies, and sometimes a whole chord can be said to have a neighboring or passing function. This is frequently the case in the different styles of popular music.

The main groove of "Psycho Killer" by the Talking Heads has a simple, somewhat hypnotic harmonic scheme that revolves around a neighbor chord. Here is Tina Weymouth's bass line with guitar chords indicated above it:



We are basically just "vamping" on A major which serves as our I chord. A more traditional (and some might say corny) approach would be to mix in a little motion to V to support our A and break up the monotony. The Heads choose to dip down to the ♭VII instead in a move that really seems like a lower neighbor to A.

VI. Passing chords

Passing chords are perhaps even more straightforward than neighbor chords. Whenever you see a progression that simply moves by step through multiple chords, it seems reasonable to deem the middle harmonies as passing, there to serve the larger motion by filling it out.

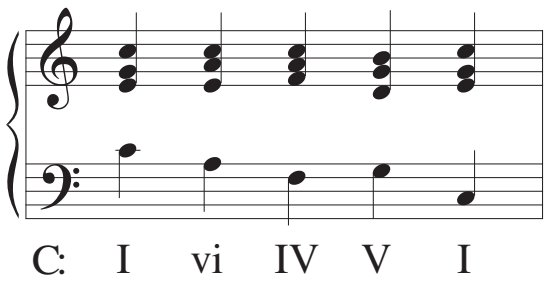
One very familiar progression that does this is i - VII - VI - V in minor keys. You might know this as "Hit the Road Jack," or maybe "Stray Cat Strut" by the Stray Cats.

```
Cm                  B♭          A♭              G
Black-and-orange stray cat sittin' on a fence,

Cm          B♭                A♭             G
Ain't got enough dough to pay the rent.

Cm        B♭          A♭              G
I'm flat broke but I don't care,

  Cm
I strut right by with my tail in the air.
```

This is a motion from the tonic in C minor down to the V, with VII and VI serving as passing chords.

VII. Chromatic third motions (aka chromatic mediants)

Third motions can be a nice way to slide through a progression, creating a subtle shift from chord to chord. In Theory I we like to practice writing this "Heart and Soul" sequence, which travels down from I to IV before retuning home.

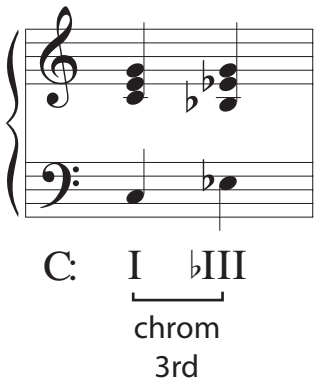


This is easy to do because there are two common tones in each chord connection. That's also why third progressions sound so smooth and gentle - they hardly change from chord to chord.

Chomatic thirds

Chromatic third motions are a little different than these subtle shifts, however. These are progression where the the chords do not fit together perfectly in the key. They will *sort of* have two common tones, but one or both with be altered with a new sharp or flat. The effect is usually much more dramatic and disorienting than a simple diatonic progression by third.

Let us consider a hypothetical move from I to ♭III in C major.

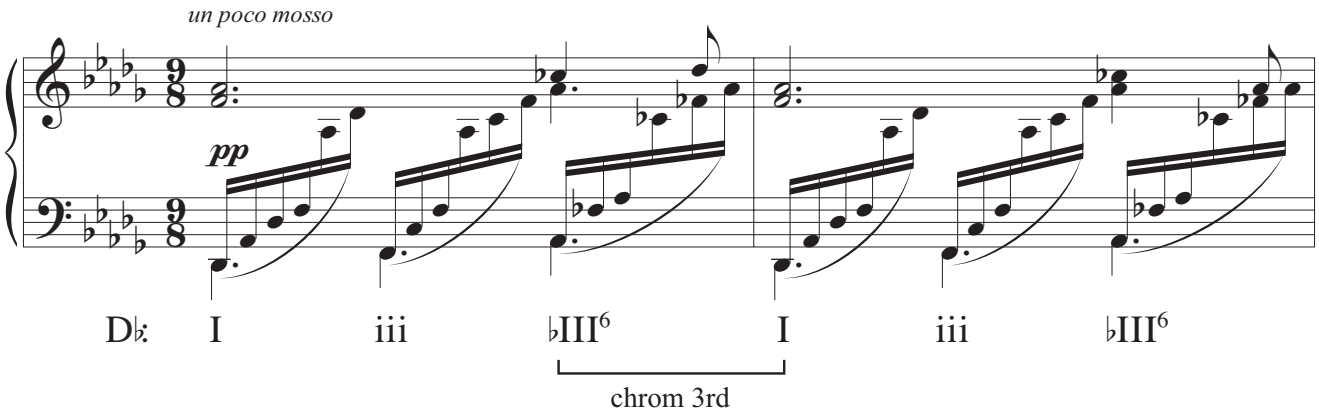


It's pretty clear how these chords should connect. They have one proper common tone, G, and one sort-of common tone that mutates a bit with an added flat (E to E♭). Because the ♭III seems to pop out of the key, this move sounds pretty dramatic and disorienting.

We'll mark these kinds of moves with a bracket and "chrom 3rd" label.

Chromatic mediants occasionally appear in Romantic and Early Modern Classical music. For example, here is a moment from Claude Debussy's popular "Clair de Lune" which is similar to our sketch.

Debussy, "Clair de Lune" from *Suite Bergamasque***, mm. 27-28**

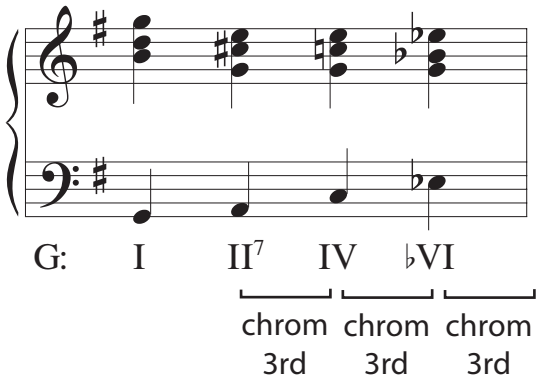


A striking example of chromatic third progression occurs towards the end of the Talking Heads' "Memories Can't Wait" [1979]. The song modulates multiple times, starting out in C-sharp minor, traveling to E major, and then to G major for the climatic final section. At the end David Byrne repeats the phrase "these memories can't wait!" over a four-chord cycle.

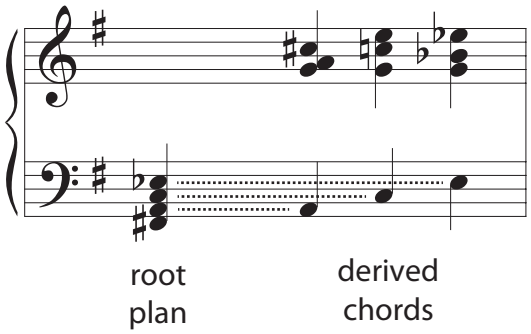
Talking Heads, "Memories Can't Wait" [1979], 2:39 to end

```
G              A7          C           E♭
     These memories can't wait!
(repeat 8x)
```

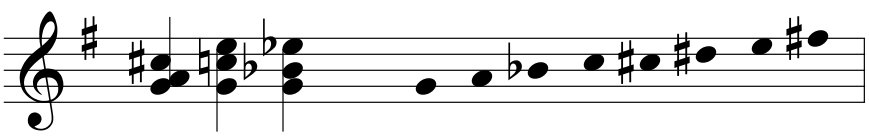
The last two chords are chromatic motions up by minor 3rd, and as we loop back around to G major that forms one last chromatic relation by major third. Here is a keyboard version of the cycle:



A student from Spring '24 (Nick Moretti) has pointed out that the last three chords in the cycle seem to be following the path of a fully diminshed seventh chord, as if they were part of the same tonal complex:



The chords mesh together to form what music theorists call the *octatonic scale*. (Jazz musicians, on the other hand, sometimes call it the "diminished scale.") It alternates whole and half steps all the way up.



IX. Modulation

Last but not least, let's consider a tune that modulates from one key to another (and back again.) On the suggestion of one of our classmates (James Reeder, Spring '25) we'll look at Paul Simon's "Kodachrome" [1973]. (James showed me an analysis at the audaciously-named howmusicreallyworks.com which is basically correct but makes a few odd choices.)

We start with the verse, which is a very conventional progression in E major.

VERSE

```
              E                E7                A
When I think back on all the crap I learned in high school
```

E: I V^7/IV IV

```
F♯m                B7              E              F♯m     B7
     It's a wonder I can think at all
```

ii V^7 I ii V^7

```
               E          E7            A
And though my lack of education hasn't hurt me none
```

I V^7/IV IV

```
F♯m                B7             E
     I can read the writing on the wall
```

ii V^7 I

Next, we get a some wild harmonies which kick off the the chorus. For now I'll just give chords names with no analysis.

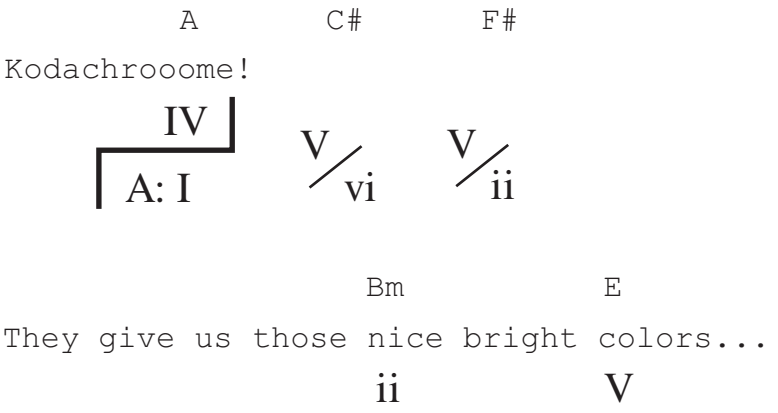
```
          A       C#       F#
Kodachrooome!
```

As we get into the chorus, it's pretty easy to hear that we are in A major. The tell-tale chord is B minor, which seems be the beginning of a ii - V - I that lands on A. I'll fill in the chorus starting on the ii.

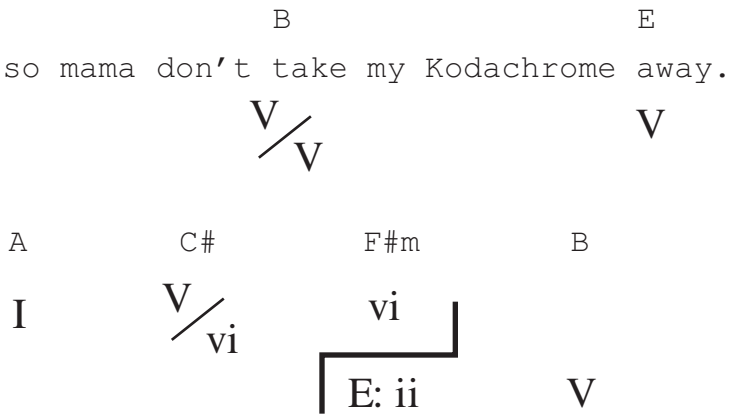
CHORUS, continued

```
                    Bm          E
They give us those nice bright colors
                  A: ii         V

                   A         D
They give us the greens of summers
                   I         IV

                  B             E
Makes you think all the world's
                 V/V            V

         A    C#  F#
A sunny day, oh  yeah!
         I   V/vi V/ii

         Bm   E
I got a Nikon camera
         ii   V

                  A    D
I love to take a photograph,
                  I    IV

               B                  E          A
so mama don't take my Kodachrome away.
               V/V                V          I
```

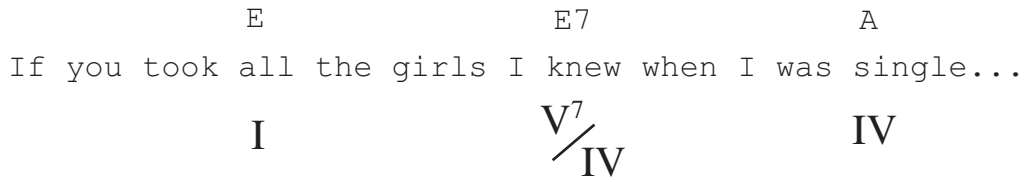
So, how does the "Kodachroooome" part get us from E to A? The two wild chords don't belong to either key, so in my opinion they aren't good options for a pivot. We've got to go back to the last chord that makes sense in both keys. Lemme add analysis and show how it leads into the chorus. We'll take our pivot right at the beginning, then go through two secondary dominants, and when we come out the other side we are definitely in A major.



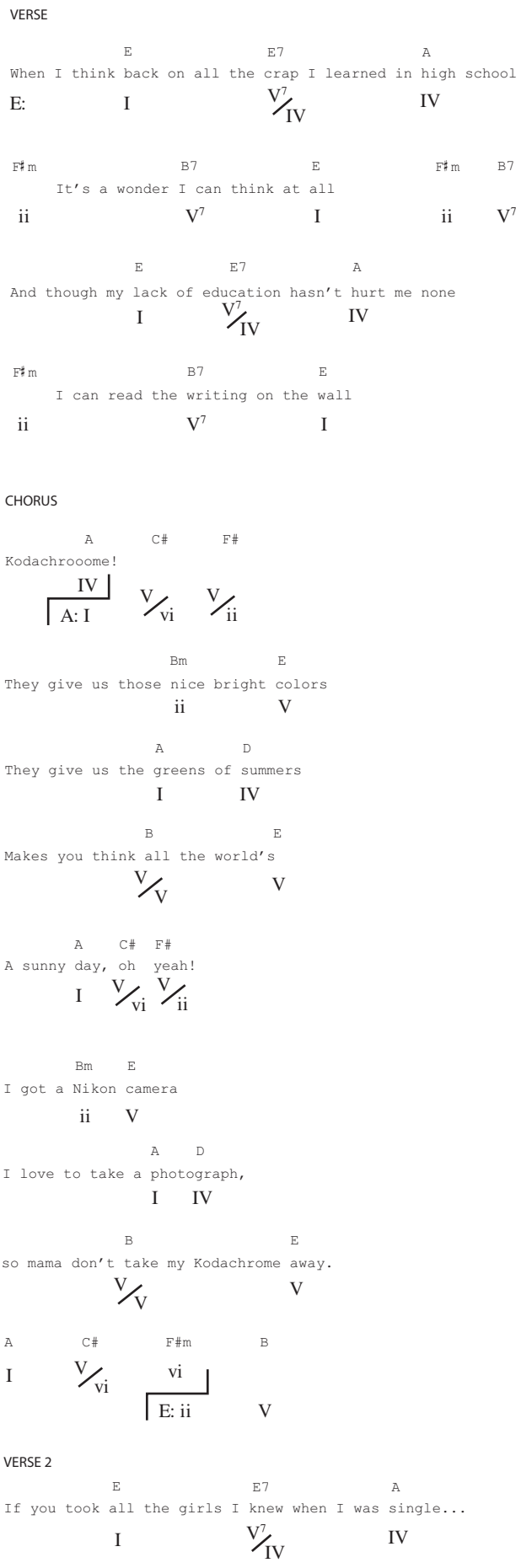
When we are done with the chorus we get a few chords that will lead us back into the verse. They are like the "Kodachrooome!" chords with one big difference - the F# chord is minor and continues on to B. I think that's our pivot on the way back - it makes reasonable sense in both keys and after that we are definitely in E.



VERSE 2



Here is the whole analysis in miniature, in case you found my stopping and starting confusing.



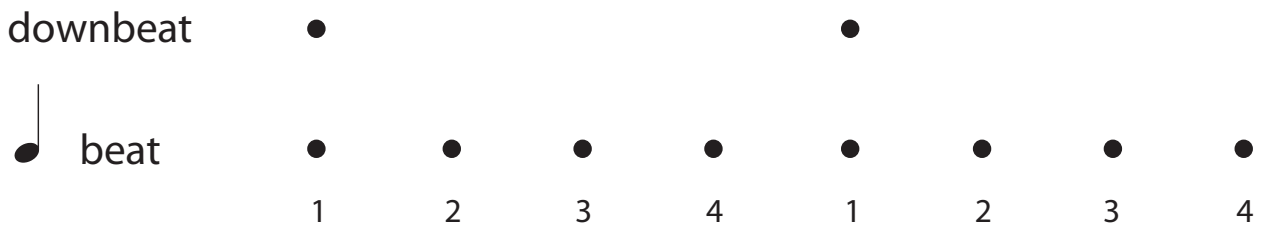
Chapter 11: Some Cool Metric Phenomena

Meter as a hierarchy of pulses

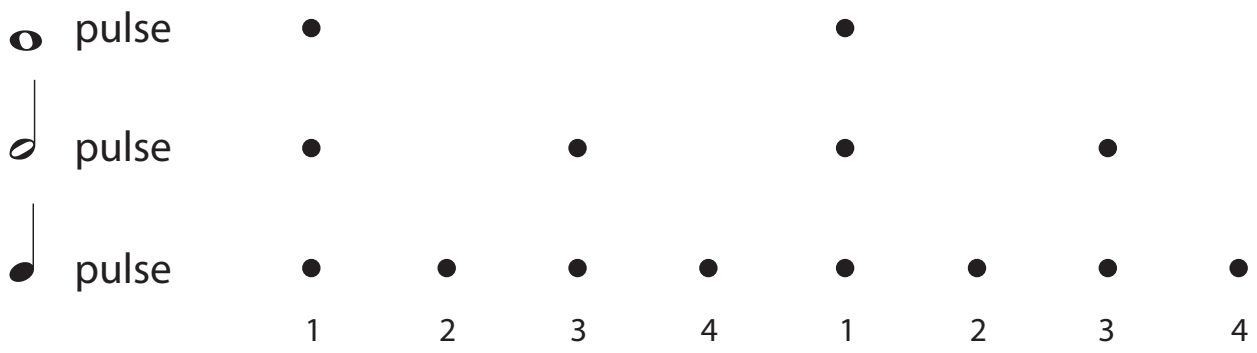
Let's start by discussing a somewhat abstract theory of meter that I find very convincing, the idea that metric patterns are really a hierarchy of regular pulses.[1]

We can start by considering our favorite time signature, 4/4. Obviously every measure of 4/4 contains four beats, which we might draw like so:

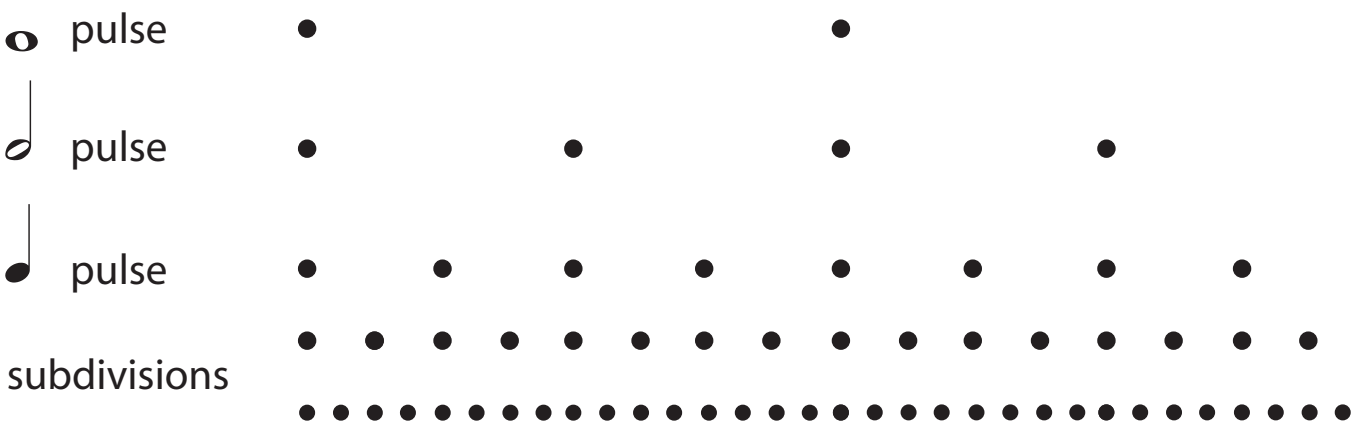
As we go from downbeat to downbeat, there is a sense that beat one is "stronger" or "more important" than the others. If the music is going fast enough it is easy to mentally focus on each downbeat and follow it to the next, as it is a really slow pulse. Lemme draw two measures with a downbeat added as an additional layer.



There is also a more subtle sense that beats one and three are stronger than two and four. In a typical rock pattern this is where we will put our bass drum sound, and it's often where we will change chords. A half-note pulse is perhaps less obvious in most music but if try to focus on it it is definitely hearable as a more laid-back beat, and it fits easily into the hierarchy.



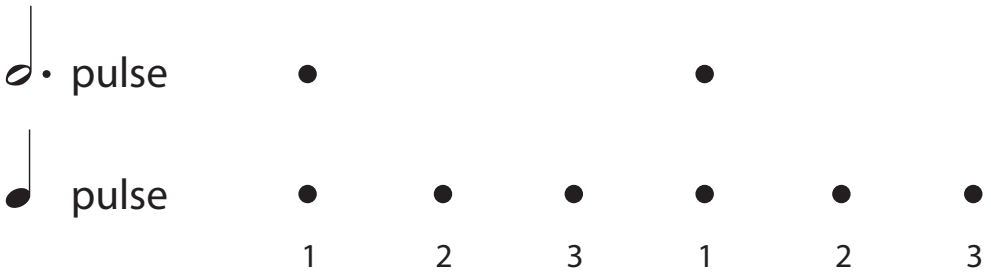
Meanwhile within the beat we might subdivide into more regular pulses that go faster. Let's do eighth notes and sixteenth notes.



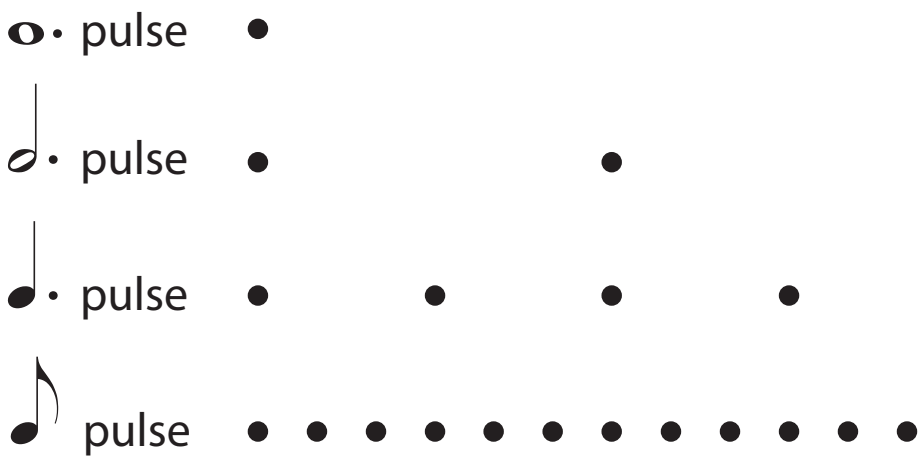
Twos and Threes

The grids I just drew featured 2:1 or *duple* relationships between layers, but of course it is also common to hear 3:1 or *triple* relationships as well.

A 3/4 groove can be diagrammed with a downbeat every three beats...



...and we hear rolling triplet subdivisions in patterns like 6/8 or 12/8.

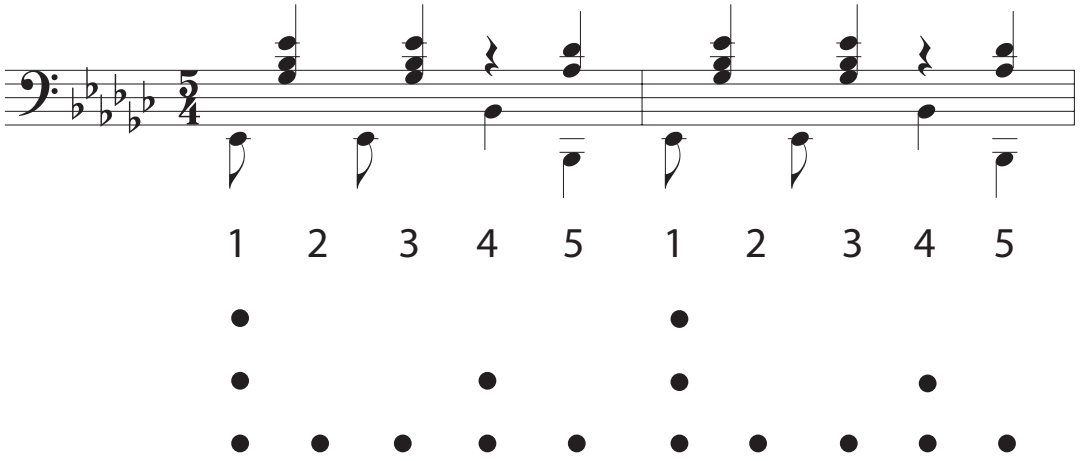


So there are a lot of possible meters that we could draw with this basic vocabulary of twos and threes. My general argument would be that **any metric pattern can be understood as a hierarchy of duple and triple subdivisions**.

Odd Meters

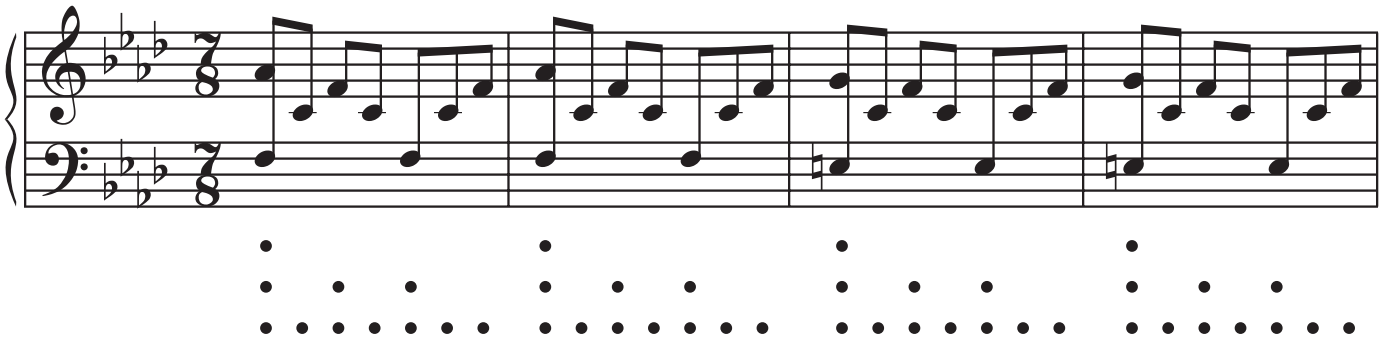
In some kinds of music you might see more uneven, tricky patterns like 5/4 or 7/8. These are called "odd meters" because they literally involve an odd number of beats and at some level they do not divide into regular pulses. However, I think these can almost always be understood in terms of twos and threes that are strung together.

Let's consider **Dave Brubeck's famous composition "Take Five,"** which has an accompanimental pattern in the piano that looks like this:

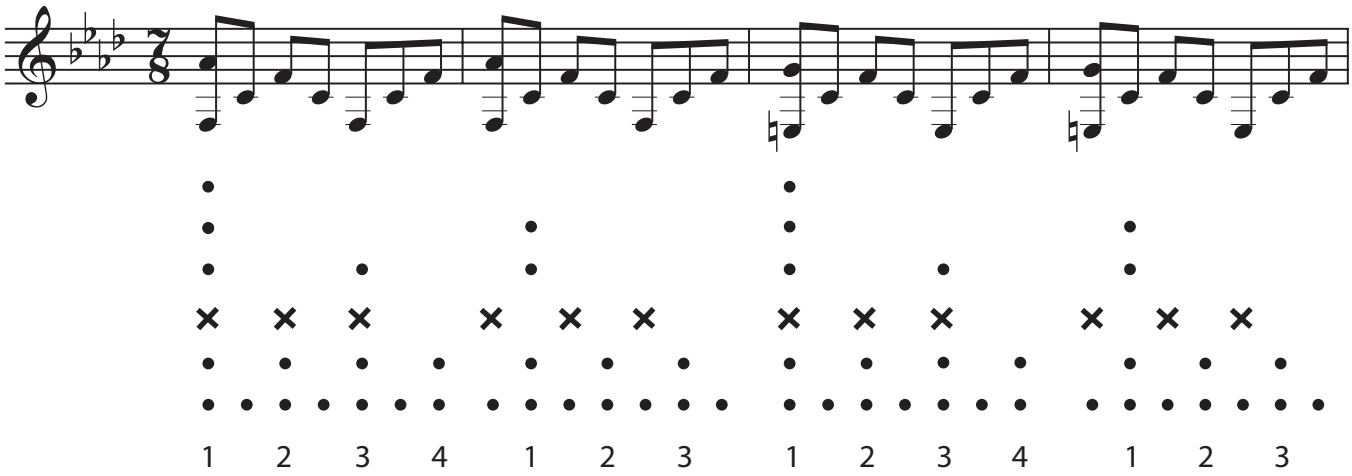


This five groove is consistently played as three beats plus two beats throughout the entire tune. While the alternation between three and two might feel slightly uneasy, the pulse at the beat level and the whole-measure level is constant and predictable, so the overall effect is gentle.

Other tunes have duple and triple groupings at the level of the subdivisions, which can create an impression of long and short beats. For example, **Radiohead's "2 + 2 = 5"** begins with a long stretch of 7/8, which is consistently played as 2 + 2 + 3 (short-short-long).



(However, since we have a preference for even beats it is tempting to smooth out this pattern into a larger 7/4 in which some of the groupings sound syncopated. I'll redraw the long and short beats as X's against a 7/4 pattern that is 4 + 3.)



Odd meters that don't repeat

Of course, not every tune locks into a pattern and sticks with it. Some are intentionally "twisty," constantly altering the succession of metric units in an unpredictable way.

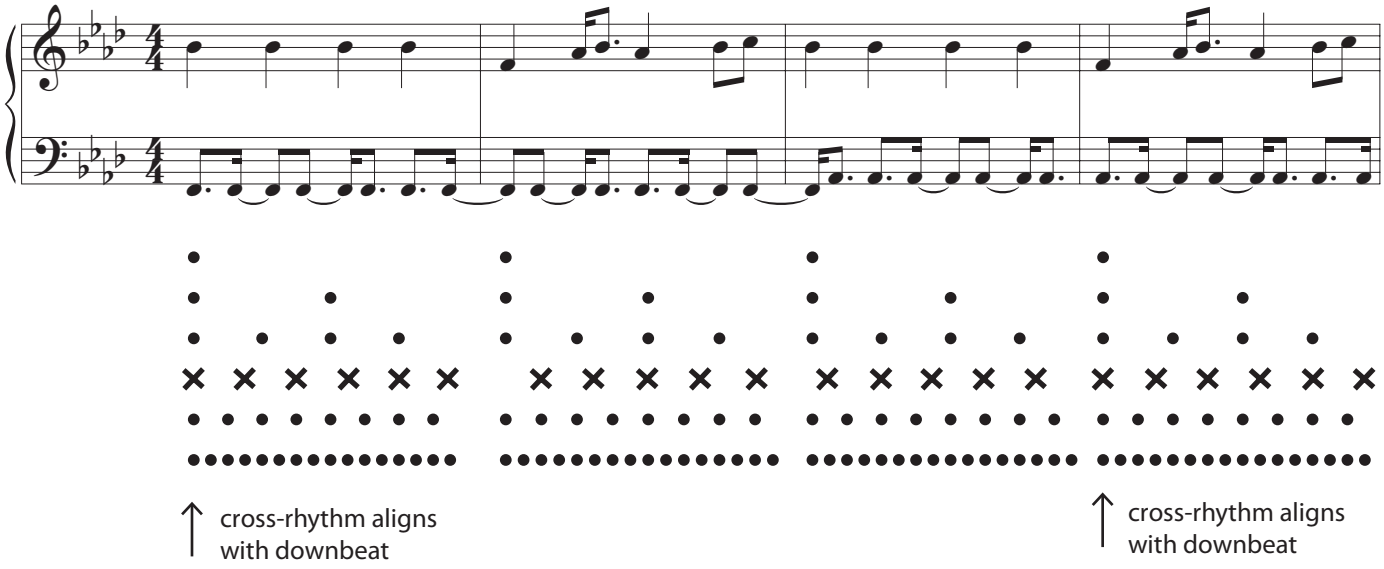
[I don't have a good example for this yet! You can hear it in Modern Classical music like Stravinsky's *L'Histoire du soldat* or Bartók's Fourth String Quartet, last movement. Perhaps I will eventually have some math rock like Don Caballero here.]

Cross-Rhythms

A cross rhythm could be defined as a pulse that is added to the texture but doesn't align neatly with our hierarchical layers. Instead, it conflicts with the overall metric scheme. I would say that a cross rhythm sounds regular and predictable but we know it is not the beat - it's a cool pattern that is laid *on top* of the beat.

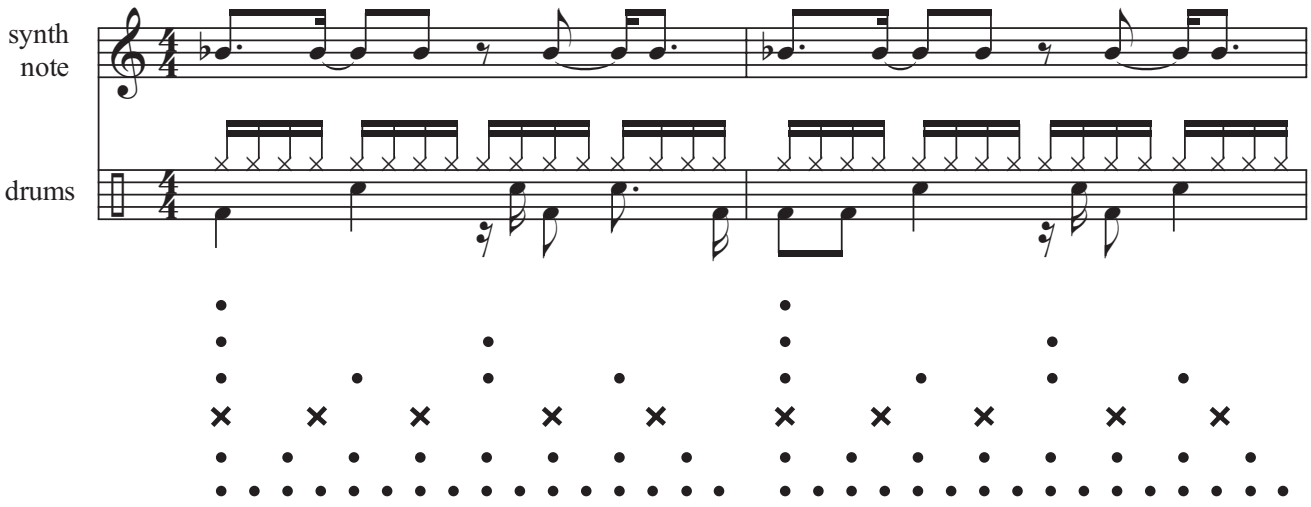
Probably the most common kind of cross rhythm rolls out a repeated grouping of three subdivisions in an otherwise duple groove. When we notate these we have to use a lot of dotted quarter notes and ties.

Let's start with a fairly extreme (but clear) example, **"Flutes" by the indie electronic pop band Hot Chip**. The track begins with a two-measure loop of chopped up vocal samples, which create a clear quarter-note beat. This is repeated a few times by itself, and then a throbbing bass enters at 0:16. The bass part makes faster groups of three against the quarter notes.



Since one measure of 4/4 has 16 subvisions in it, our three pattern takes some time to land back where it started. 16 is not divisible by three, nor is 32, so we have to wait for three whole measures (i.e. forty-eight 16th notes) before our pattern hits another downbeat.

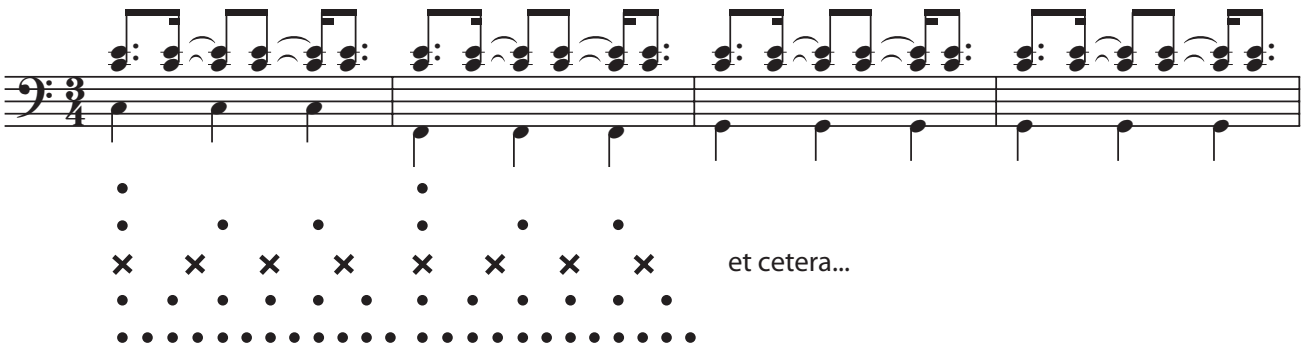
Since these rolling threes fit so poorly into a 4/4 scheme musicians will often "reset" the pattern rather than letting it continue indefinitely. For an example let's look at the beginning of **Nas's "NY State of Mind"** which starts a pattern on the downbeat but resets it on the "and" of three.



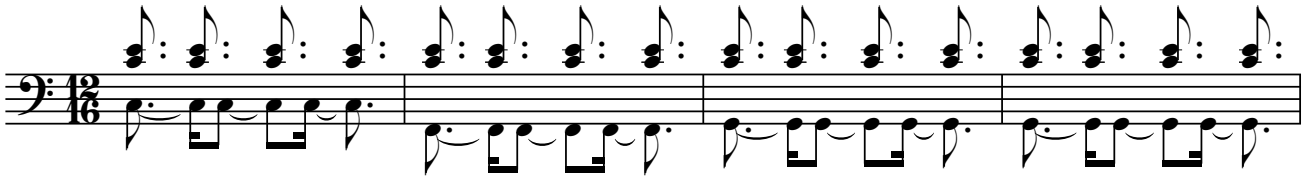
I think that this qualifies as a cross-rhythm even though it resets. As long as it establishes a predictable pattern for three or four pulses that's good enough for me. :)

Promoting a cross-rhythm to a pulse

One interesting trick in recent music production is to introduce a cross rhythm early on in a track and then later reorganize the metric scheme so that it becomes an essential part of the beat. Let's look at **"Fake Empire" by The National**, which starts with this pattern on piano:



The pattern presents an ambiguous "four against three" conflict that runs for the whole song. In the beginning the low notes in the piano sound strong, and one's intial interpretation would probably be that this is in a slow three, like I've notated it above. However, once Matt Berninger starts to sing the four pattern starts to feel more prominent, and we might start to hear a 12/8 context. (I've notated it in 12/16 below to capture the intricate feeling of the track.)

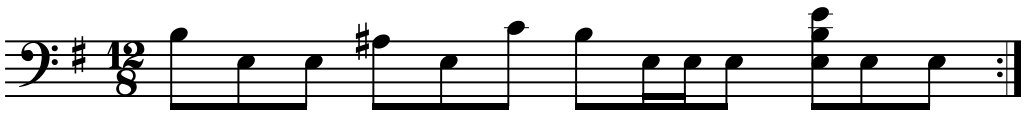


Eventually the drums and bass kick in at 1:15 and reveal a strong beat in three, in the configuration I notated at the outset. In my experience this is a surprising reveal, because we've been hearing it in four for a full minute or so.

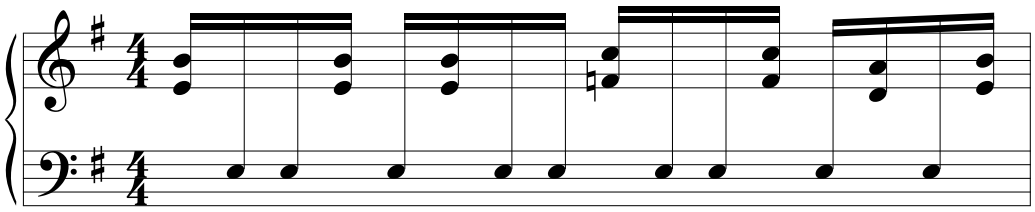
Metric Modulation

I would say that the cross-rhythms trick is a form of *metric modulation*. It's a method of converting one metric pattern to another by changing some rhythmic values but keeping others constant. Sometimes we see this kind of transformation without the foreshadowing of cross rhythms - a composer can jump from one pattern to a related pattern that transforms what is happening in an interesting way.

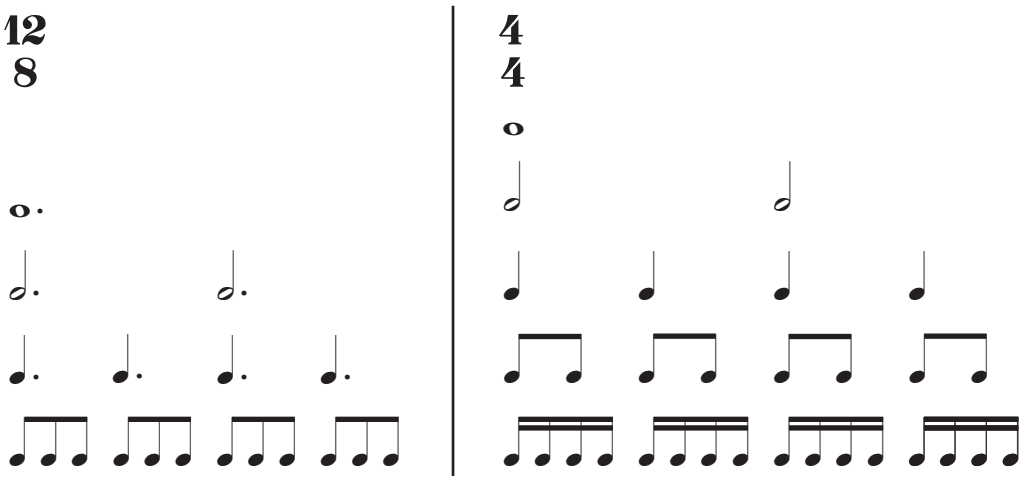
Faith's No More's "Surprise! You're Dead" modulates back-and-forth between a 12/8-style groove and a 4/4 in a dramatic shift that is surprising but also very logical. It begins with a repeated guitar riff that can be written with triplet subdivisions. I'll notate it in 12/8:

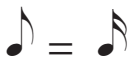


However, at 0:37 it shifts to a 16th-note-driven 4/4 with a slower beat.[2]



What has happened is that the subvisions have remained the exact same length. The eighth notes in my top sketch become the sixteenth notes in the bottom sketch. Because a beat is now four subdivisions instead of three, the BPM is cut by 25%, from 127 beats per minute to about 95. We can draw a map of this transition with the metric scheme on each side and notes on how one groove converts to the other.



 subdivisions are constant

 beat length increases (4:3)

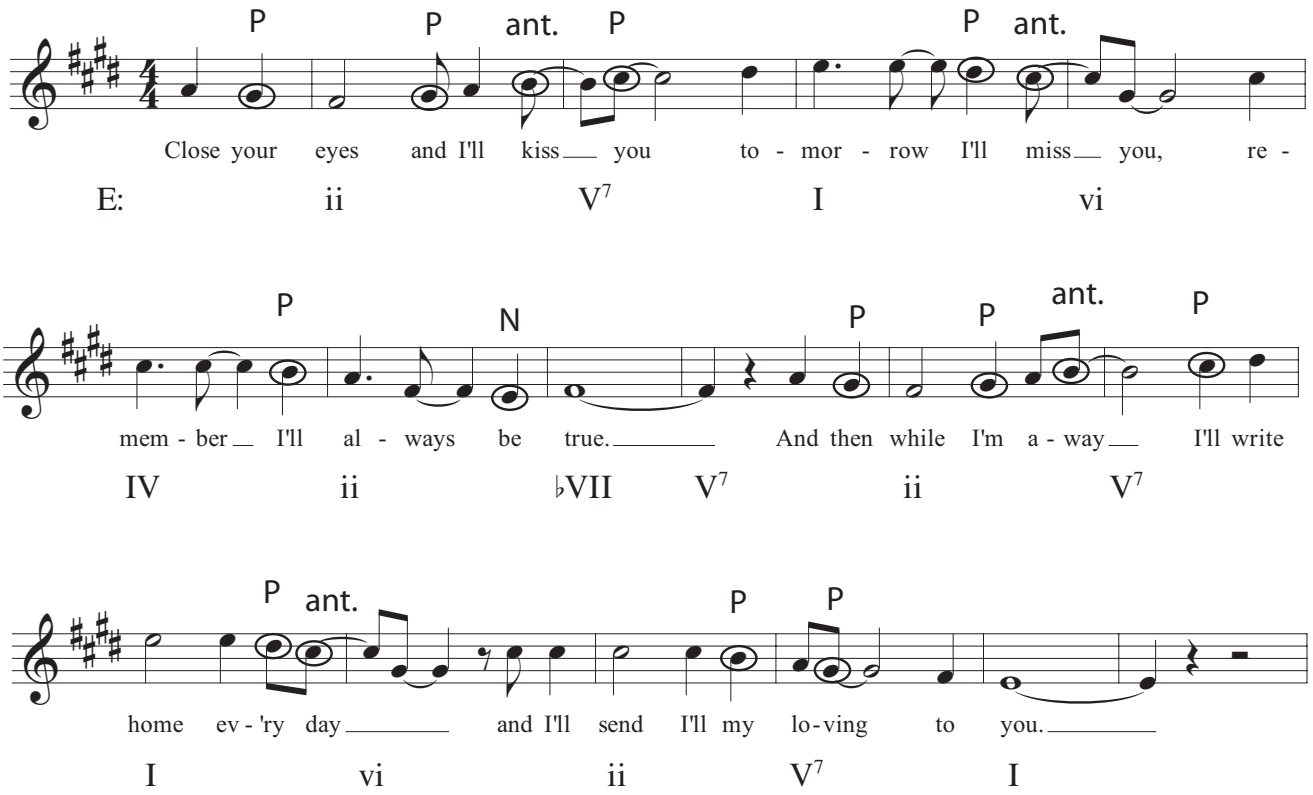
~127bpm ~95bpm beat rate decreases (3:4)

This Faith No More tune flips back and forth between the left side of my map and the right side. However, it is possible to create a piece of music that transforms in multiple stages, arriving at a final tempo that is in a complex relationship to where we started out.

Chapter 12: Melody and Harmony in Pop Songs

In this chapter we'll consider the way melodies fit together with the harmonies that support them. We'll look at one pop tune that is fairly straightforward and one that exhibits a complex layering known as the "melodic-harmonic divorce."

First, let's consider a relatively simple tune, **the Beatles' "All My Loving."** The chord progression is close to our standard flowchart, with a lot of traditional ii-V7-I action in there and some moves out to vi. The only really unusual progression is on "remember I'll always be true" as Lennon and McCarthy cycle through a series of descending thirds from IV to ii to the subtonic (bVII) and finally to a half cadence on V.



Looking at the melody we see a lot of stepwise motion through these harmonies, which gives the impression that this is a tune that simply falls "naturally" into place.

There are still some interesting details that stick out, however. In particular, there are several moments like "and I'll kiss you" where the melody seems to jump ahead of itself, making an anticipation of a chord tone before the barline and shifting a passing tone to the "and" of one.

The Melodic-Harmonic Divorce

However, some popular tunes have melodies that do not seem to follow the harmonies that go with them. The "melodic-harmonic divorce" is a relatively recent term that pop and rock theorists have coined to describe songs in which the melody and harmonies seem to form two conflicting layers.

Let's look at a tune in which the melody and chords don't fit together in a straightforward way, **"Teenage FBI" by Guided by Voices.**[1]

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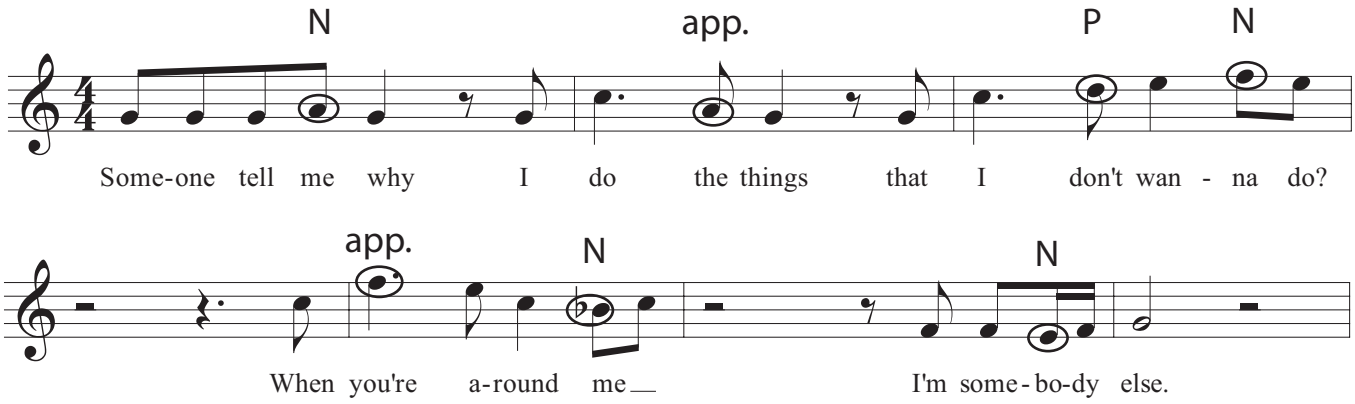
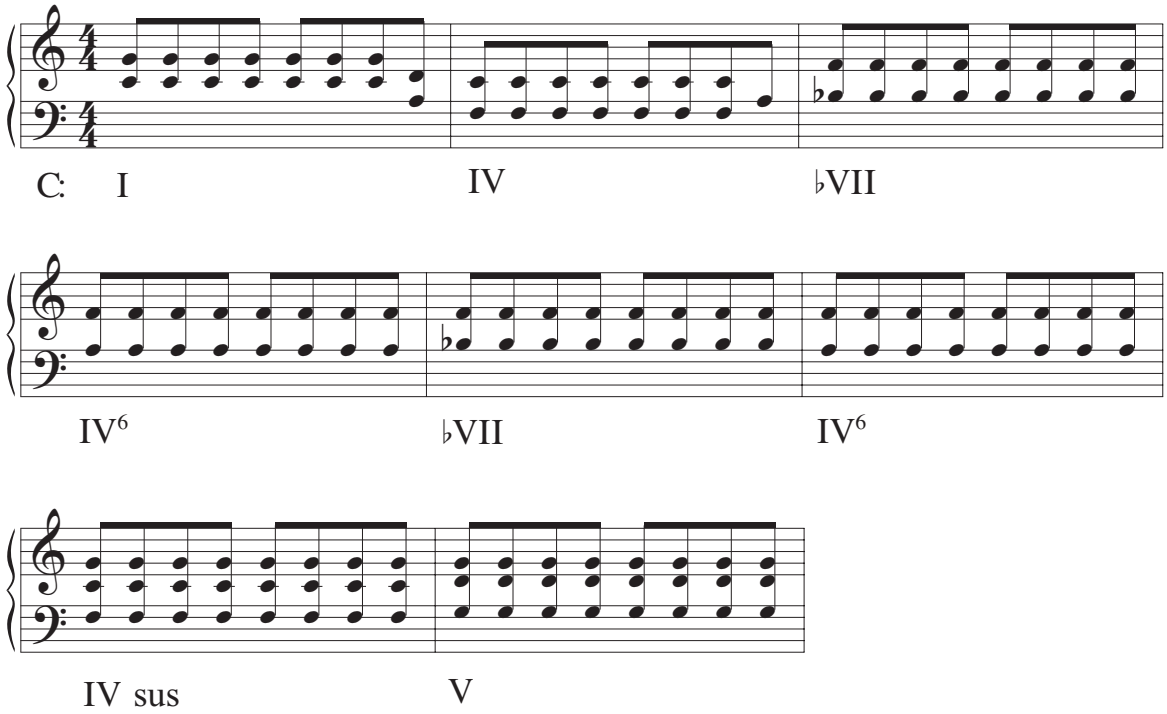
[1]I'm going to refer to the original version of the song, which was released on a single in 1997 and collected in the compilation *Human Amusement at Hourly Rates*. This version is recorded with the guitars tuned very sharp, closer to D♭ major than C. However, other versions suggest that the song is conceived in C.

Let's start by looking at the melody in isolation. Sometimes it is possible to judge which notes are essential and which are decorations by the rhythm and shape of a line. This is obvious in the first few notes of the song, which are mostly G's with one brief move up to A. A is very likely to be an upper neighbor to G!

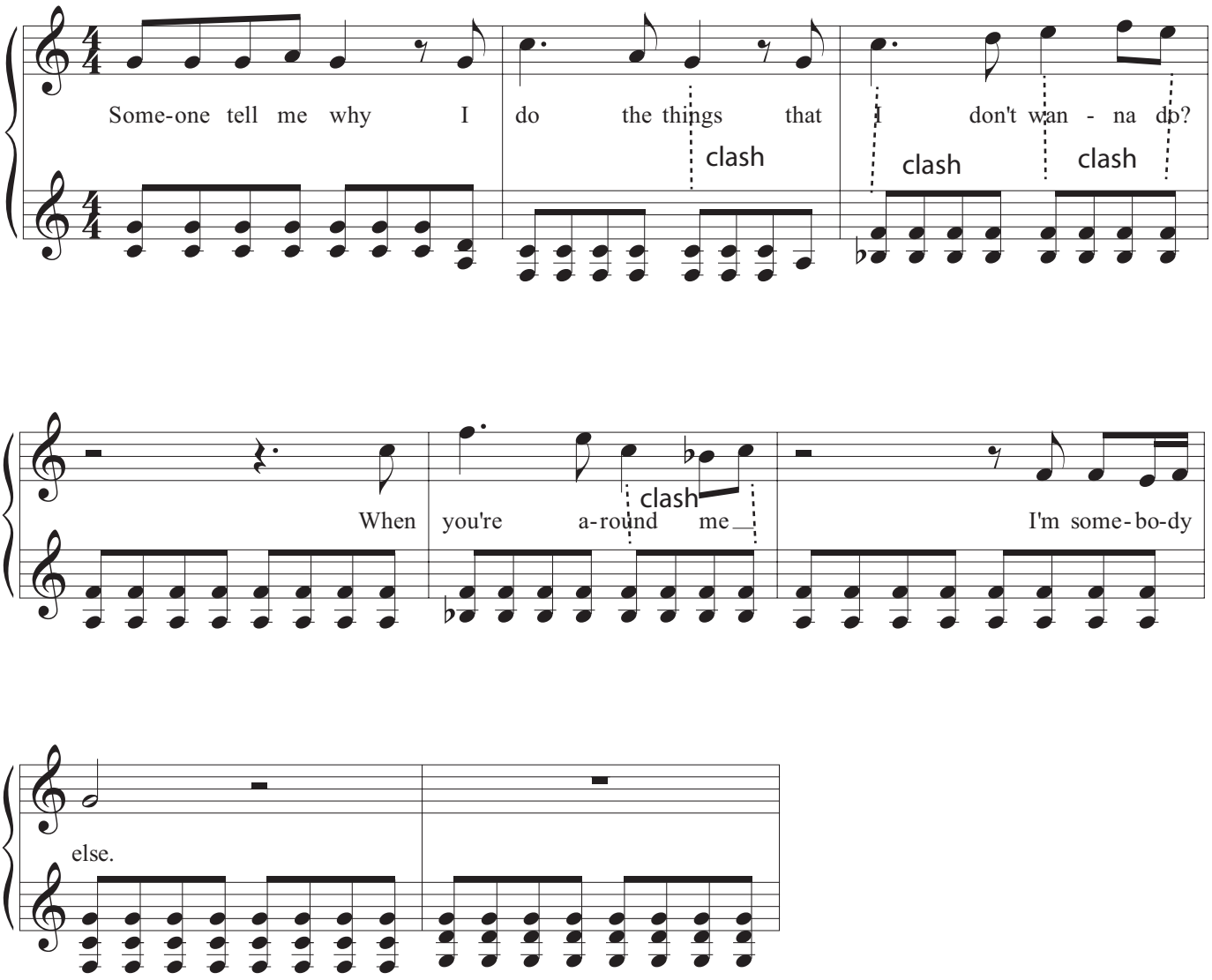
Some other judgements are a bit harder to explain. The G-C jump on "I do..." implies that both notes are part of a chord, because you can't "orphan" decorations (i.e. all of the traditional non-chord-tones attach to some other note.) The E at the end of the phrase is probably a chord tone on similar grounds (that you don't just leave a dissonance hanging.) As we work our way through the first seven measures it feels reasonable to interpret the melody as mostly emphasizing the tonic triad - there are maybe some alternate choices one can make but I think this is the most persuasive interpretation.



Now lets look at the underlying chords. The opening part of the song includes a single guitar part playing some sparse intervals. The implied harmonies include a lot of IV as well as the ♭VII (which we might call "IV of IV").



When you put both parts together, you can see that they don't really correspond. A lot of the tones in the melody that we thought were structural actually conflict with the guitar part. This creates an overall feeling of tension that is only released at the end of the passage, as we pull up to V on "I'm somebody else".



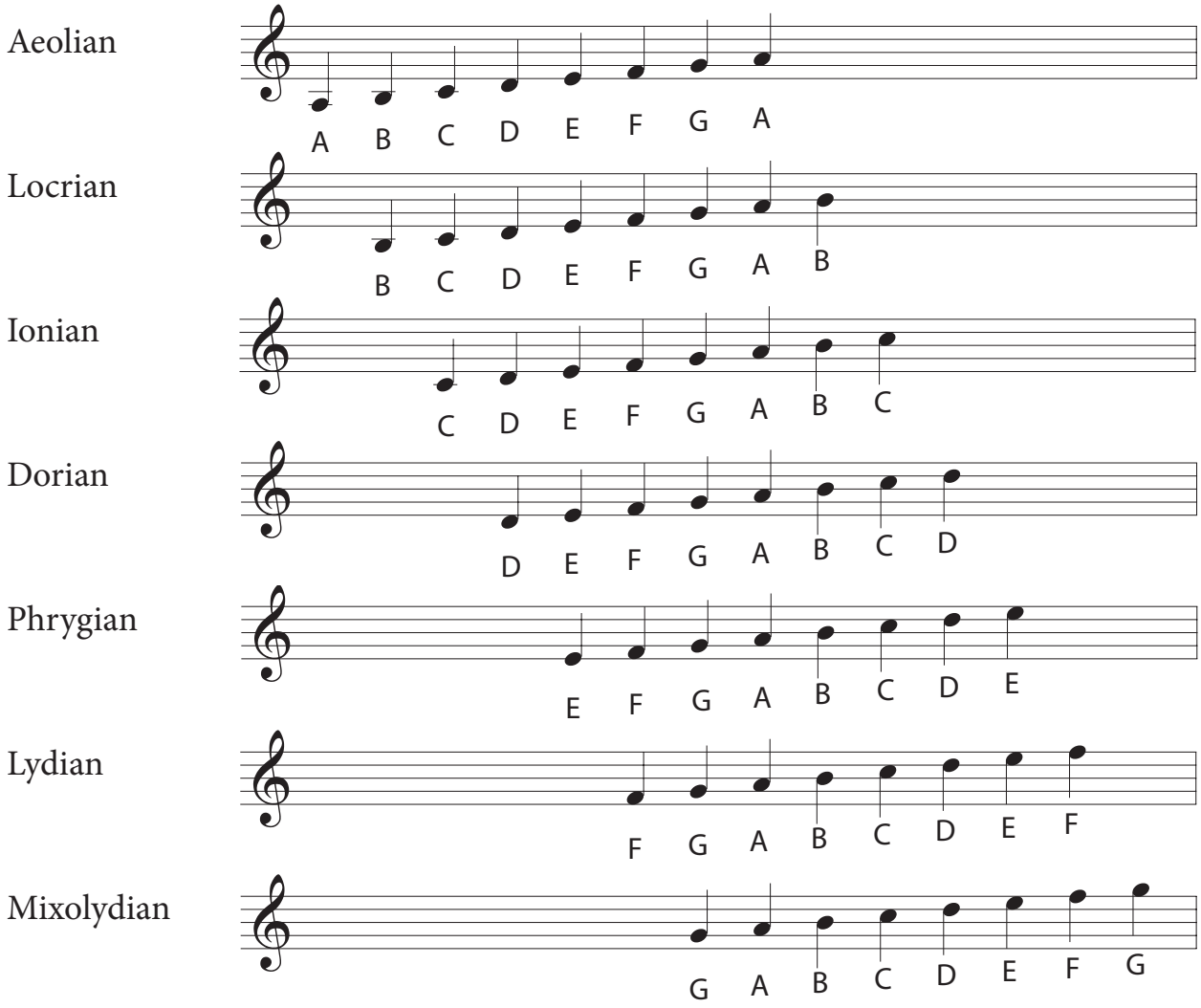
This is the typical arrangment with the so-called melodic-harmonic divorce - we often see **a melody line that sticks to the tonic while backing instruments play other chords**.

Chapter 13: New Scales and Collections

Modes

European musicians in the Middle Ages lived mostly in a white-note world. They used sharps and flats very sparingly, and didn't have anything like the system of 12 major and minor keys we have today. Instead, they achieved tonal variety through the *church modes*, a system of seven different scales that can be derived from the white notes of the piano. They gave them somewhat fanciful geographical names inherited from the Greeks.

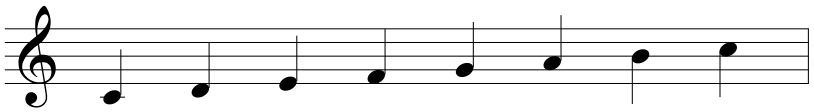
We already know that you can derive the major scale by playing from C to C on the piano, and the "natural" minor is A to A. Imagine more scales generated from every possible segment of the white keys.



Let's organize these scales another way. We'll transpose them so that they all start on C. We'll group all of the major-type scales together (i.e. those with a major $\hat{3}$) and also all of the minor-type scales (with the minor $\hat{3}$).

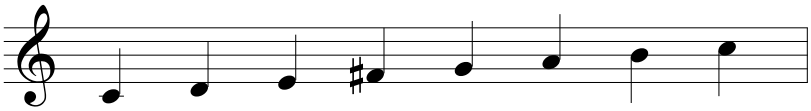
Majorish Modes

C Ionian



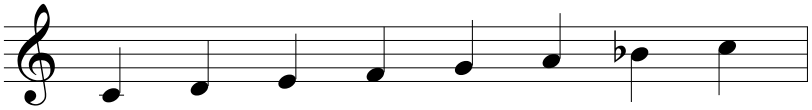
This is the standard major scale.

C Lydian



Major with $\sharp\hat{4}$

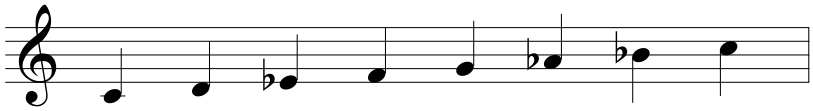
C Mixolydian



Major with $\flat\hat{7}$

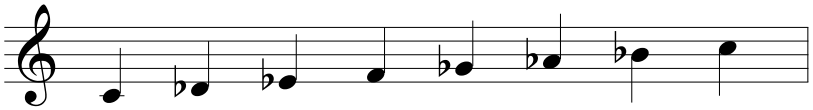
Minorish Modes

C Aeolian



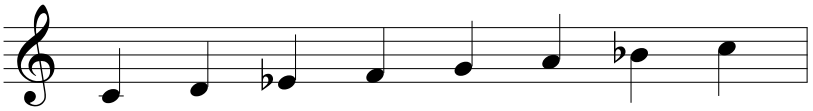
This is the standard "natural" minor scale.

C Locrian



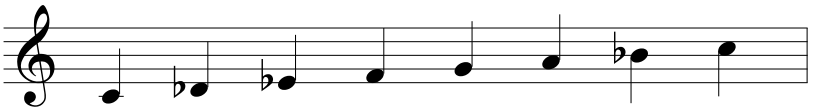
Minor with ♭$\hat{2}$ and ♭$\hat{5}$. Kind of its own thing, really, like a diminished-type scale.

C Dorian



Minor with a raised $\hat{6}$. A fresh and modern-sounding minor!

C Phrygian



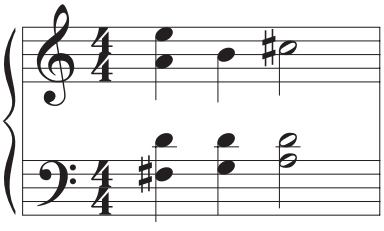
Minor with a ♭$\hat{2}$.

The Diatonic Collection

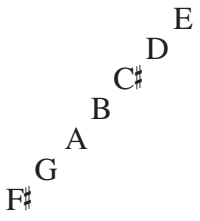
In a sense these modes are all "the same notes," just ordered differently. An E Phrygian scale uses the white keys, organized in such a way that E is the tonic or "home note." F Lydian is the same notes with F as tonic instead.

Sometimes the tones are more loosely organized, so that you can't say for sure where the scale begins and ends. We can talk about this group of notes in general as the *diatonic collection*. A diatonic collection doesn't have to be actual white notes, of course, just any group of tones that can be organized into one of these modes.

Some modern-sounding chords, tonic is unclear



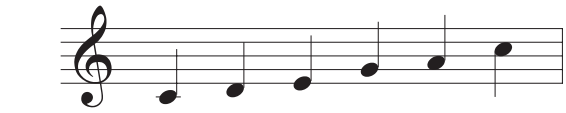
gather them together into some rotation of D major



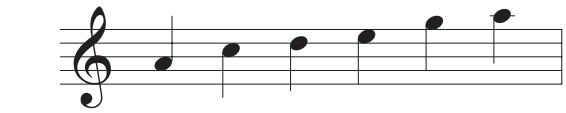
= a diatonic collection

The Pentatonic Scale

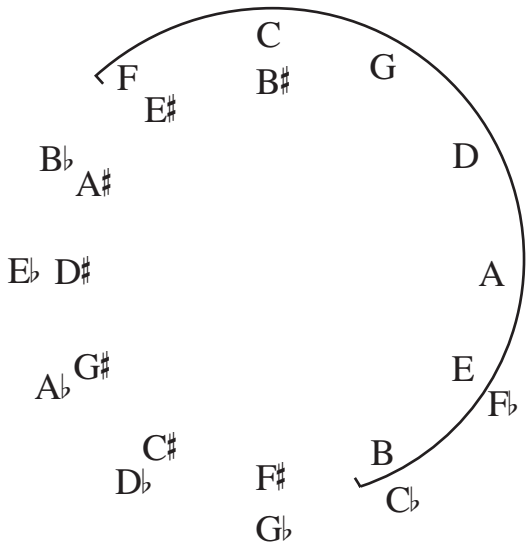
The pentatonic scale only has five notes in it. It occurs in many different kinds of music throughout the world, and is particularly useful for rock and jazz improvisers. Perhaps the easiest way to remember it would be to build a major triad, fill in the $\hat{2}$, and add a $\hat{6}$. This is sometimes called the *major pentatonic*.



However, the scale can be rotated around other ways to make more shapes. Here is a version that emphasizes the minor triad instead. It is sometimes called the *minor pentatonic*.

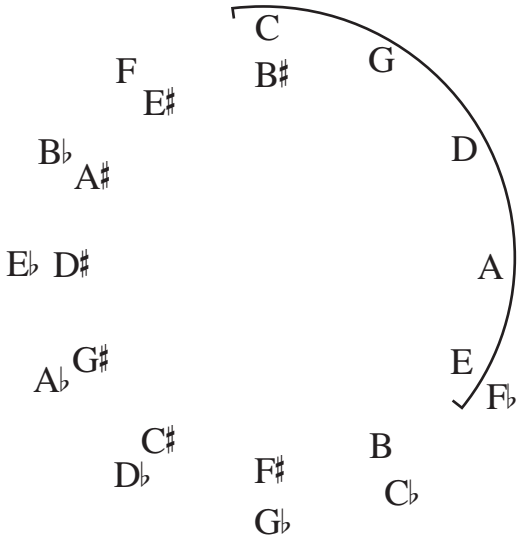


I think the relationship between the diatonic collection and the pentatonic is very interesting, for two reasons. First, imagine a circle of fifths that contains all possible pitches. (We'll make it curl around on itself to get the different enharmonic spellings.) The C major scale can be generated by taking a segment of this circle. If we shift this zone one position clockwise it generates G major (picking up F#), and if we shift it the other way it makes F major (picking up Bb).



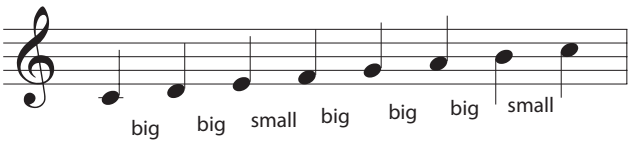
Pitch circle in fifths, C major collection is selected as a 7-note segment.

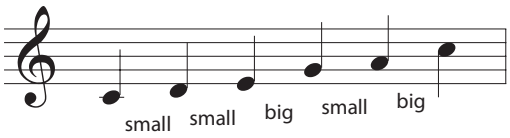
Interestingly, the pentatonic scale is also a smaller segment from this circle, taking five notes instead of seven.



Pentatonic collection is selected as a 5-note segment.

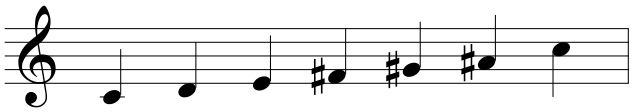
Also, perhaps more perceptually relevant is the way both scales fill up musical space. A diatonic scale fills up the octave with two different intervals - a big one (a whole step) and a smaller one (the half step). The pentatonic also has two interval sizes, but the large is a minor third and the small is a whole step. It's like the diatonic scale "swelled up," leaving room for fewer notes.





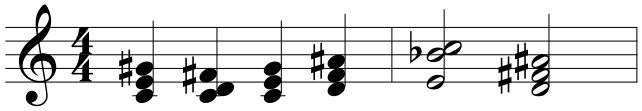
The Whole-Tone Scale

The whole tone scale couldn't be simpler to construct, because it's all whole tones all of the way up.



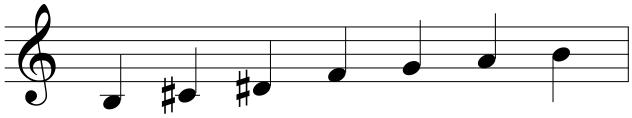
(OK fine, we do need to skip one letter name, which creates the illusion of a gap where there is none. Since this is a non-traditional scale there are no fixed rules on how to spell it. Otherwise it is very simple, though!)

This creates an unusual sound, because it can only make augmented triads and other "whole-toney" shapes. (One other familiar chord we might see is the dominant seventh with no fifth in it, like C-D-F#.)



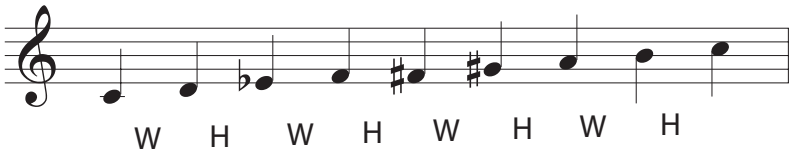
Some whole-tone sonorities

In a sense there are only two whole tone collections. There is the one that might start on C, like we've been using so far, and there is the other one that could start on B and contains all of the leftover notes, B-C#-D#-F-G-A.



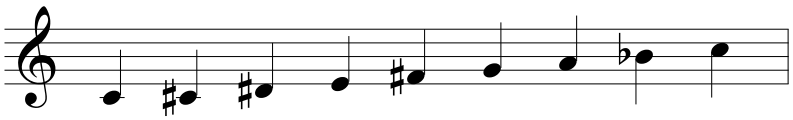
The Octatonic Scale

The octatonic scale alternates whole steps and half steps in a repeating pattern. It manages to cram eight notes into the octave (hence the name "octatonic.")



octatonic starting on C ("whole-half" version)

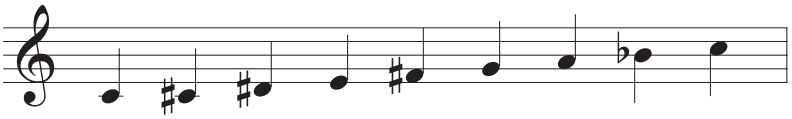
Jazz musicians sometimes call this "the diminished scale." You could distinguish between two forms of the scale, the "whole-half" one (which might start C-D-Eb, like my example above) and the "half-whole" version (which might start C-C#-D#).



octatonic starting on C ("half-whole" version)

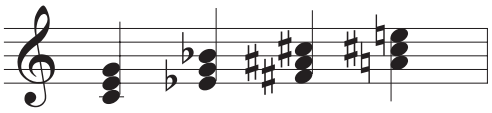
Just as there are a limited number of possible whole-tone scales, there are only three octatonic collections. One somewhat elegant way to specify which one you are talking about would be to say "the octatonic collection that contains C and D," as opposed to the one that contains C and C# or the one that contains C# and D. Obviously you can use any two adjacent notes in the collection as a reference.

The octatonic scale is particularly useful for composers because it can make a large number of triads and other familiar sonorities.

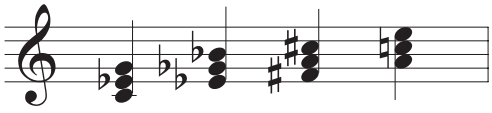


The C-C♯ octatonic collection can produce...

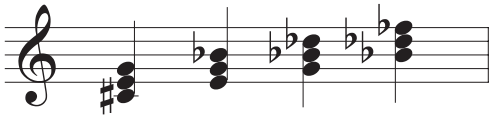
major triads



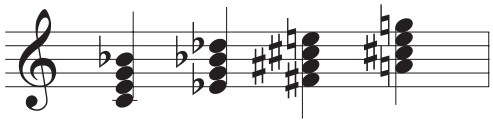
minor triads



diminished triads



dominant 7th chords



...and much, much more!

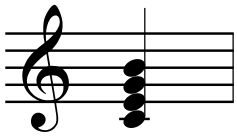
Chapter 14: Extended Harmonies in Jazz

Jazz harmony often involves complex sounds that go well beyond the seventh chords we are used to. In this chapter we'll look at how to understand chord symbols with ninths, elevenths, and thirteenths.

The basic seventh chords

First, let's look at the seventh chords we already know about, and how jazz charts indicate them. It is pretty much the norm in jazz that *all* chords include a seventh.

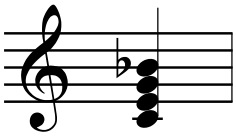
major 7th



C maj7 or C$^{\Delta 7}$

Major triad with major seventh. Jazz indicates this with "maj7" or the triangle symbol.

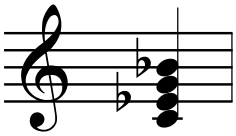
dominant 7th



C^{7}

Major triad with minor seventh. Like in classical theory, this is sort of the default type, indicated with just a 7.

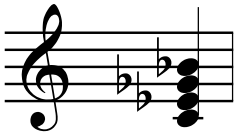
minor 7th



C-^{7} or Cm^{7}

Minor triad with minor seventh. Jazz likes to use a lowercase m or a dash for minor - it generally doesn't use lowercase letters.

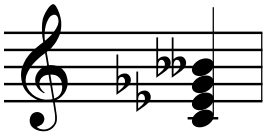
half-diminished 7th



Cm$^{7}_{\flat 5}$

Diminished triad with minor seventh. In jazz this is indicated as a minor seventh chord "with flat five" because jazz doesn't have the circle with a slash.

fully diminished 7th



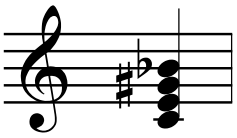
C$^{\circ 7}$

Diminished triad with diminished seventh. Indicated with the circle, like in classical theory.

Two more seventh chords

In addition you might see a few other four-note chords.

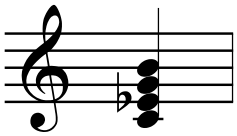
augmented 7th



C^{+7}

Augumented triad with a minor seventh.

minor-major 7th

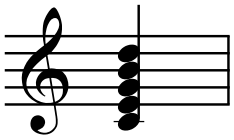


$C\text{-}^{\Delta 7}$

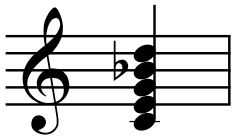
This one is rare but it does pop up occasionally. Minor triad with a major 7th added to it.

Ninths

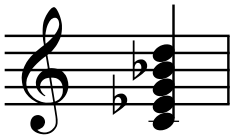
The idea of the ninth is that we continue to stack up thirds until we get to a $\hat{2}$ that is lofted up above the other notes. You can make a major ninth chord, a dominant ninth or a minor ninth - all of these imply the regular seventh chords we know about with this new, extra note stacked on top.



$C^{\Delta 9}$



C^{9}

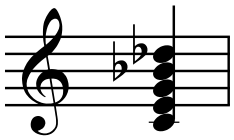


Cm^{9}

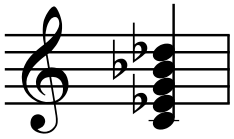
Altering the ninth

You can also have a flat nine or a sharp nine, or even both! Obviously these are a half step lower or higher than the "plain" ninth.

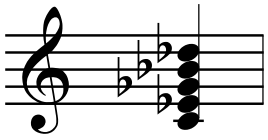
The flat nine sounds good when it's added to a dominant, minor, or half-diminished seventh. Note that in my chord symbol I'll list a seventh with "extra" flat nine - this is typically how you indicate altered tones.



C$^{\flat 9}_{7}$

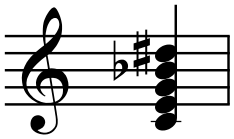


Cm$^{\flat 9}_{7}$



Cm$^{\flat 9}_{7\ \flat 5}$

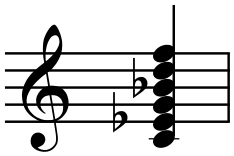
The sharp nine is a nice addition to the dominant seventh - it adds a "blues third" to the chord and creates a funky clash.



C$^{\sharp 9}_{7}$

The Eleventh

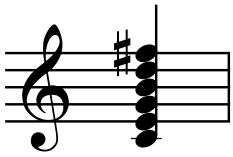
The eleventh lofts a $\hat{4}$ up onto the top of the chord. The most likely candidate for a regular, unaltered 11th would be the minor 11th, which implies a full chord with seventh and ninth as well.



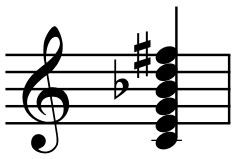
Cm^{11}

Sharp Eleventh

The sharp eleventh is, of course, a half step higher than the regular version, conceptually a tritone plus octave above the root. This altered chord extension is surprisingly versatile - it sound great added to a major ninth or dominant ninth...

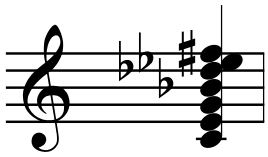


C$^{\sharp 11}_{\Delta 9}$



C$^{\sharp 11}_{9}$

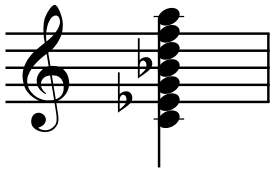
AND it goes well with the altered ninths. If you use a sharp eleventh and want an "altered" sound it makes sense to list both the flat 9 and sharp 9, since they fill out the chord nicely.



C$^{\flat 9\ \sharp 11}_{7\ \sharp 9}$

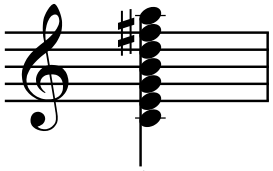
Thirteenth

Last but not least is the thirteenth, which adds a note that is conceptually a major sixth plus octave above the root. The 13th sounds great atop a minor-type chord:

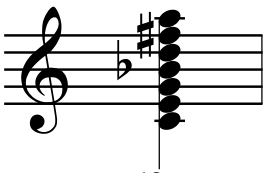


Cm^{13}

...and it also goes well with major and dominant-type chords with #11.



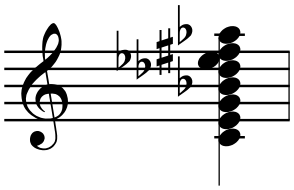
C$^{\Delta 13\ \sharp 11}$



C$^{13\ \sharp 11}$

Flat Thirteenth

The flat thirteen is a minor 6th plus octave above the bass. This goes well with our other "altered" tones. Here is a dominant-type chord with the works:



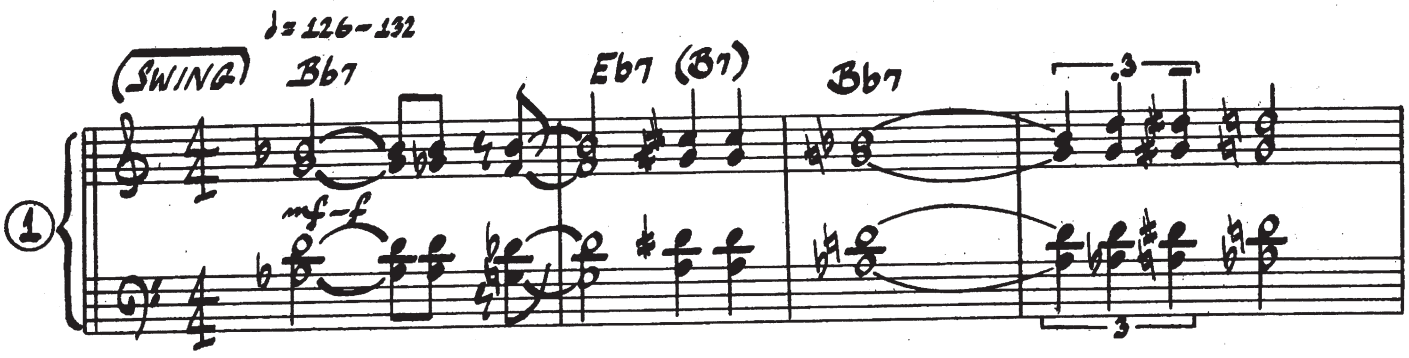
These aren't the only combinations

In this discussion I've focused on extended harmonies that I thought sounded typical for jazz. By experimenting I'm sure you can find other cool combinations of tones, including some that really don't sound good at all.

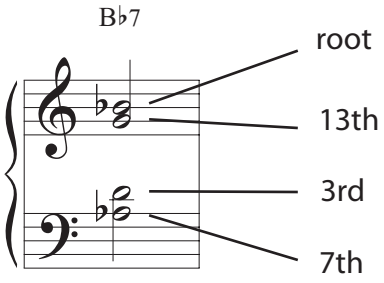
Jazz voicings in real life

Of course real jazz pianists and guitarists don't usually play their chords in neat stacks like this. Just like with our classical-style progressions a jazz musician will rotate notes around into different configurations and strategically omit tones to get their own unique sound.

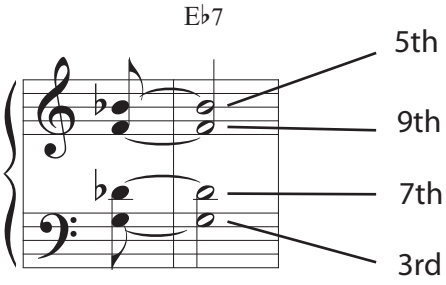
Here is a quick excerpt from Jamey Aebersold's "Jazz Piano Voicings" (Jamey Abersold Jazz, 1980), p. 27. Aebersold recorded a volume of backing tracks for students to play along to, and then had someone transcribe all of his voicings, so we can see exactly what he plays. These are the first three chords in a B♭ blues, with a little extra embellishment here and there.



Examining the first voicing, we see that Aebersold tucks a 13th in with more basic tones, and he puts the root of the chord on top.



When he slides into the E♭7 he keeps a common tone on top, connects his thirds and sevenths, and adds a ninth into the right hand.



One can spend a lifetime exploring different ways to incorporate interesting chromatic tones and dissonances into your music, in your compositions, chordal "comping" and improvised solos.

Chapter 15: Atonal Theory

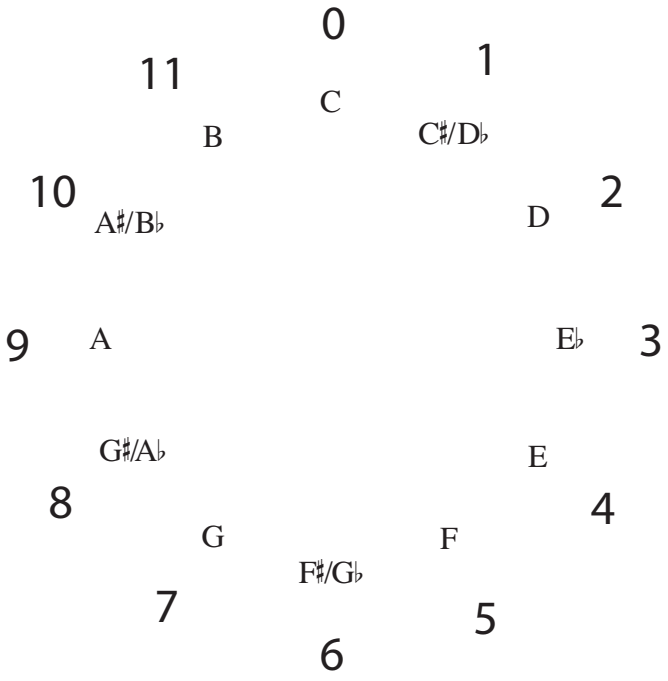
In the first few decades of the 20th century we started to see music that didn't seem to follow the traditional principles of tonal organization. Composers like Schoenberg, Stravinsky, Webern and Boulez often experimented with unfamiliar, dissonant combinations of notes and intentionally avoided any sense of an underlying scale or tonic note.

Theorists responded to this new music with a new set of analytic tools, a new "atonal" or "post-tonal" theory. While musical trends have perhaps moved beyond the moment that inspired these ideas, these Modernist concepts can still be stimulating and productive for today's musician.

In this chapter we'll learn a few simple tools that can help us grapple with any combination of notes we might encounter.

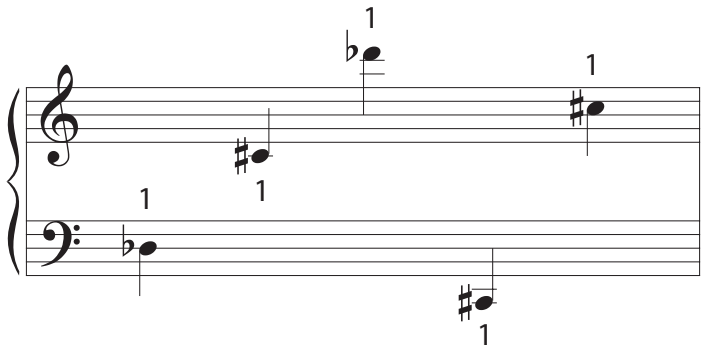
The circle of pitch classes

The central idea of post-tonal theory is the **pitch class**. Instead of referring to notes by their traditional names, we are going to give them each a number. Every possible note can be organized onto a circle of twelve integers, from zero to eleven.



In doing this we have imposed two abstractions on the world of traditional theory. First, we've imposed **enharmonic equivalence** - we've decided that C# really is the same thing as Db and we call them both 1. I imagine many students will be relieved to know that we are finally in a world where the way a note is spelled does not matter.

Also, in more radical move we've imposed **octave equivalence.** The pitch class 1 stands for every possible version of this note in any octave.



Now, in labeling all of these pitches as PC 1 we are not denying that the notes all sound different. Everybody agrees that they do! But, we are looking for an underlying continuity in these sounds, and I think we can also agree that they also sound very much like "the same note" in different octaves.

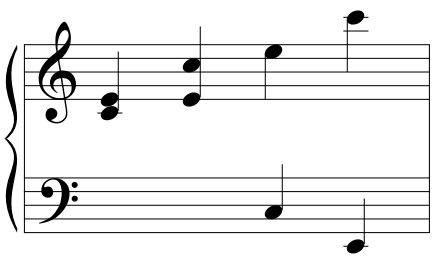
Really, in tonal theory we make octave generalizations as well - we agree that the notes C, E and G represent a C major triad, regardless of where they are distributed on the musical staff. (They can be bunched together or spread out, placed high or low, and rotated around in different ways.) Post-tonal theory is looking for the same kind of underlying structure that is provided by chords and scales in traditional theory.

Interval Classes

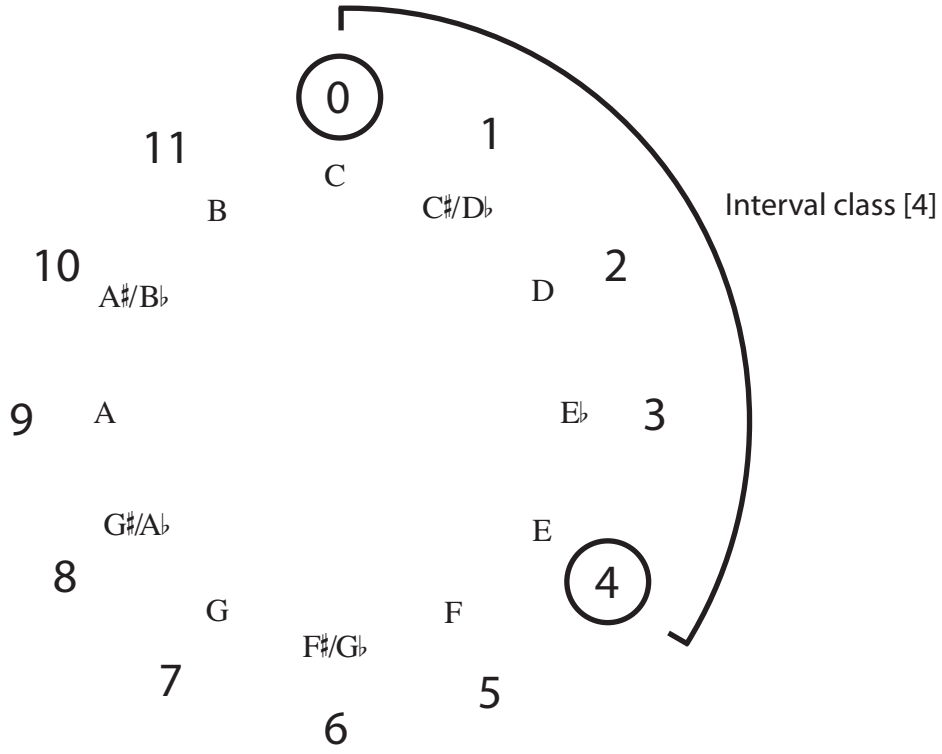
Putting all of our notes onto the circle of PCs creates a situation in which the surface of the music is abstracted into something sleeker and simpler. One useful result of this abstraction is the interval class.

Let's consider an interval between C and E. You already know that this relationship could be expressed as a major third, it could be flipped around into a minor sixth, or these intervals could be expanded even further by adding octaves.

A bunch of possible C-E intervals.



On the circle of pitch classes, however, the intervals all look the same.



In order to describe the interval made by C and E we take the shortest distance on the circle, which is its *interval class*. There are only seven interval classes, and this is an example of IC [4].

[0] - unisons, octaves, multiple octaves

[1] - minor 2nds, major 7ths and their octave multiples

[2] - major 2nds, minor 7ths etc.

[3] - minor 3rd, major 6ths

[4] - major 3rd, minor 6ths

[5] - perfect 4ths, perfect 5ths

[6] - the tritone

Interval Vectors

One way to describe the sound of any combination of notes is to take an inventory of interval classes in it, which is known as an *interval vector*.

For our first "atonal" chord to analyze, let's select the last combination of notes from Schoenberg's Six Little Pieces for Piano, first movement:

Taking a pitch-class inventory, we've got 10, 4, 8 and 3. If we gather them together and put them in numerical order it makes a pitch-class set of (3, 4, 8, 10).

Next we go through every pair of notes in the set, and note the IC made by each one.

(3, 4, 8, 10)
↑ ↑
[1]

(3, 4, 8, 10)
↑ ↑
[5]

(3, 4, 8, 10)
↑ ↑
[5]

(3, 4, 8, 10)
↑ ↑
[4]

(3, 4, 8, 10)
↑ ↑
[6]

(3, 4, 8, 10)
↑ ↑
[2]

Then we gather the information together into a little array, like so:

<1, 1, 0, 1, 2, 1>

This indicates that there is one IC [1], one IC [2] and so on, down the line. With a smattering of almost everything in there this chord is pretty complex.

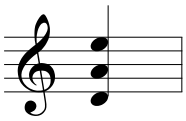
Set Classes and "Prime Form"

Another way to study a collection of notes is to boil it down to a referential version called a *set class*. In order to find the set class we can plot our pc set on the circle, and we'll look for the shape that has the *smallest outside interval* and is *most-packed to the left*. We'll consider both the version that is presented as-is in the music and it's upside-down inversion in our search for the truly optimal referential form.

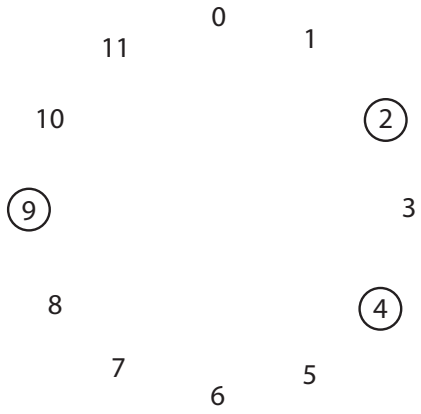
It sounds complicated but it's not that hard! If you work it out on the circle of PCs it is very visual.

1. Organize your notes into a PC set

Let us consider a cool little stack of fifths, like this:

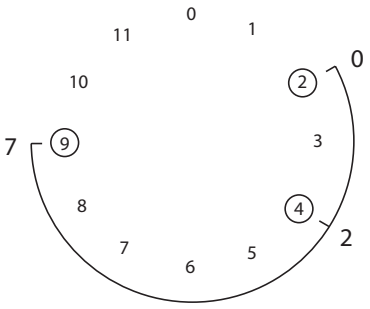


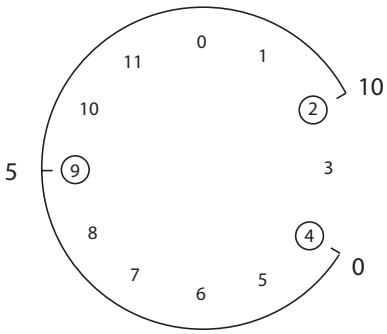
Plotting this on the circle automatically organizes it into a PC set for us.

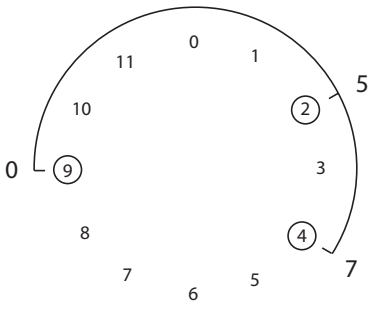


2. Look for the shape with the smallest outside interval

We are looking for the version of the set with the shortest distance from first note to last. If we do this visually on the circle we can see that there are three ways to trace the shape - we can count from D to A, E to D, or A to E. We'll count each shape starting with zero.





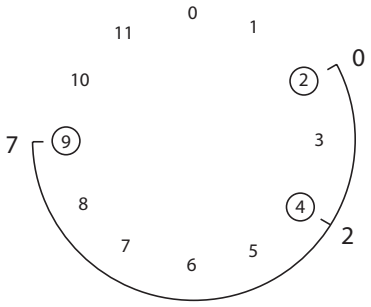


If you don't like the visual approach you could say that we are rotating the set around and transposing each version down, mod 12, so it starts on zero. (2, 4, 9) becomes [0, 2, 7], (4, 9, 2) becomes [0, 5, 10], and (9, 2, 7) becomes [0, 5, 7].

Obviously we can reject the version that starts on E, since [0, 5, 10] has a giant outside interval of 10. That leaves us with two candidates for the prime form of our set, [0, 2, 7] and [0, 5, 7].

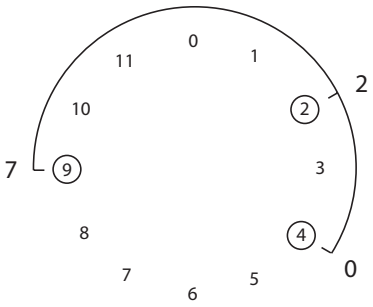
3. Look for the "most packed to the left."

If there is more than one possible version of the set with the same outside interval, we select the one that has the lower number as you read from left to right. [0, 2, 7] has a two in the second position, whereas [0, 5, 7] has a five, so the first version wins. [0, 2, 7] is our prime form.



4. Consider the inverse shape

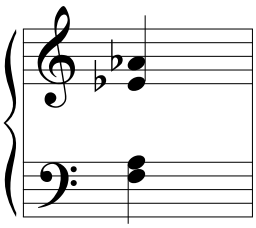
We also want to consider the inverse shape to see if maybe that is more packed to the left. This is just a matter of counting our shapes in the counter-clockwise direction. Here we see that the inverse shape starting on E also produces [0, 2, 7]. That's not better that the right-side-up version, so we won't worry about it. Our answer is still [0, 2, 7]!



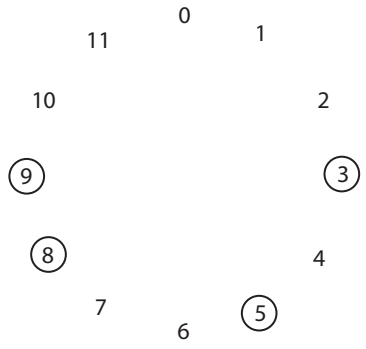
(If you are programming a computer to do this or something you'd have to multiply each pc in the set by -1, take that mod 12, and rotate that set around to find the best prime form. I think you should make it easy on yourself and just look at the circle counter-clockwise.)

Lets consider one more set that does trigger this rule #4.

Let's try the same operation on a funky F^7 chord with sharp 9.

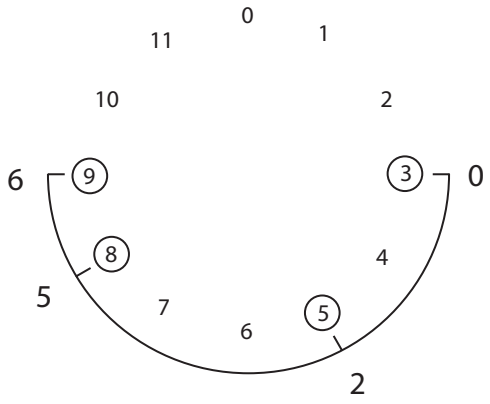


1) We put it on the circle



2) We find the smallest outside interval

Here is it visually obvious that there is only one span with the smallest outside interval. A quick-and-dirty shortcut you can use in this case is to scan for the biggest gap in the circle and take the opposite.

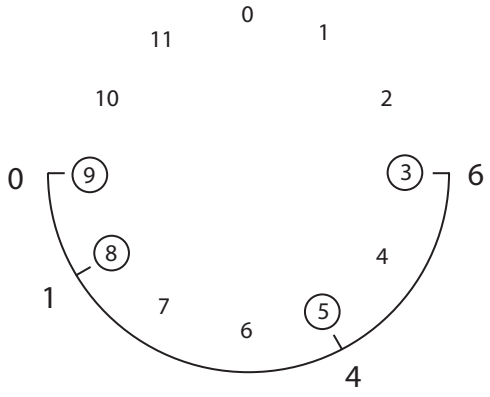


2) Select most packed to the left

We've only got one viable option so far, [0, 2, 5, 6], so I guess this step is not relevant.

3) Consider the inverse shape

BUT when you take the same span and consider it counter-clockwise, you see that [0, 1, 4, 6] is more packed to the left. [0, 1, 4, 6] is our answer!



The table of all set types

Here is a list of all possible set types in the dodecaphonic system. The labels before each set were introduced by theorist Allen Forte. This is an easy way to doublecheck your work after you figure out a set type in prime form - if it's on this list, you probably did it right! In our examples we found a 3-9 [0, 2, 7] and a 4-Z15 [0, 1, 6, 7].

2-1 [0, 1]
2-2 [0, 2]
2-3 [0, 3]
2-4 [0, 4]
2-5 [0, 5]
2-6 [0, 6]
3-1 [0, 1, 2]
3-2 [0, 1, 3]
3-3 [0, 1, 4]
3-4 [0, 1, 5]
3-5 [0, 1, 6]
3-6 [0, 2, 4]
3-7 [0, 2, 5]
3-8 [0, 2, 6]
3-9 [0, 2, 7]
3-10 [0, 3, 6]
3-11 [0, 3, 7]
3-12 [0, 4, 8]
4-1 [0, 1, 2, 3]
4-2 [0, 1, 2, 4]
4-3 [0, 1, 3, 4]
4-4 [0, 1, 2, 5]
4-5 [0, 1, 2, 6]
4-6 [0, 1, 2, 7]
4-7 [0, 1, 4, 5]
4-8 [0, 1, 5, 6]
4-9 [0, 1, 6, 7]
4-10 [0, 2, 3, 5]
4-11 [0, 1, 3, 5]
4-12 [0, 2, 3, 6]
4-13 [0, 1, 3, 6]
4-14 [0, 2, 3, 7]
4-Z15 [0, 1, 4, 6]
4-16 [0, 1, 5, 7]
4-17 [0, 3, 4, 7]
4-18 [0, 1, 4, 7]
4-19 [0, 1, 4, 8]
4-20 [0, 1, 5, 8]
4-21 [0, 2, 4, 6]
4-22 [0, 2, 4, 7]
4-23 [0, 2, 5, 7]
4-24 [0, 2, 4, 8]
4-25 [0, 2, 6, 8]
4-26 [0, 3, 5, 8]
4-27 [0, 2, 5, 8]
4-28 [0, 3, 6, 9]
4-Z29 [0, 1, 3, 7]
5-1 [0, 1, 2, 3, 4]
5-2 [0, 1, 2, 3, 5]
5-3 [0, 1, 2, 4, 5]
5-4 [0, 1, 2, 3, 6]
5-5 [0, 1, 2, 3, 7]
5-6 [0, 1, 2, 5, 6]
5-7 [0, 1, 2, 6, 7]
5-8 [0, 2, 3, 4, 6]
5-9 [0, 1, 2, 4, 6]
5-10 [0, 1, 3, 4, 6]
5-11 [0, 2, 3, 4, 7]
5-Z12 [0, 1, 3, 5, 6]
5-13 [0, 1, 2, 4, 8]
5-14 [0, 1, 2, 5, 7]
5-15 [0, 1, 2, 6, 8]
5-16 [0, 1, 3, 4, 7]
5-Z17 [0, 1, 3, 4, 8]
5-Z18 [0, 1, 4, 5, 7]
5-19 [0, 1, 3, 6, 7]
5-20 [0, 1, 5, 6, 8]
5-21 [0, 1, 4, 5, 8]
5-22 [0, 1, 4, 7, 8]
5-23 [0, 2, 3, 5, 7]
5-24 [0, 1, 3, 5, 7]
5-25 [0, 2, 3, 5, 8]
5-26 [0, 2, 4, 5, 8]
5-27 [0, 1, 3, 5, 8]
5-28 [0, 2, 3, 6, 8]
5-29 [0, 1, 3, 6, 8]
5-30 [0, 1, 4, 6, 8]
5-31 [0, 1, 3, 6, 9]
5-32 [0, 1, 4, 6, 9]
5-33 [0, 2, 4, 6, 8]
5-34 [0, 2, 4, 6, 9]
5-35 [0, 2, 4, 7, 9]
5-Z36 [0, 1, 2, 4, 7]
5-Z37 [0, 3, 4, 5, 8]
5-Z38 [0, 1, 2, 5, 8]
6-1 [0, 1, 2, 3, 4, 5]
6-2 [0, 1, 2, 3, 4, 6]
6-Z3 [0, 1, 2, 3, 5, 6]
6-Z4 [0, 1, 2, 4, 5, 6]
6-5 [0, 1, 2, 3, 6, 7]
6-Z6 [0, 1, 2, 5, 6, 7]
6-7 [0, 1, 2, 6, 7, 8]
6-8 [0, 2, 3, 4, 5, 7]
6-9 [0, 1, 2, 3, 5, 7]
6-Z10 [0, 1, 3, 4, 5, 7]
6-Z11 [0, 1, 2, 4, 5, 7]
6-Z12 [0, 1, 2, 4, 6, 7]
6-Z13 [0, 1, 3, 4, 6, 7]
6-14 [0, 1, 3, 4, 5, 8]
6-15 [0, 1, 2, 4, 5, 8]
6-16 [0, 1, 4, 5, 6, 8]
6-Z17 [0, 1, 2, 4, 7, 8]
6-18 [0, 1, 2, 5, 7, 8]
6-Z19 [0, 1, 3, 4, 7, 8]
6-20 [0, 1, 4, 5, 8, 9]
6-21 [0, 2, 3, 4, 6, 8]
6-22 [0, 1, 2, 4, 6, 8]
6-Z23 [0, 2, 3, 5, 6, 8]
6-Z24 [0, 1, 3, 4, 6, 8]
6-Z25 [0, 1, 3, 5, 6, 8]
6-Z26 [0, 1, 3, 5, 7, 8]
6-27 [0, 1, 3, 4, 6, 9]
6-Z28 [0, 1, 3, 5, 6, 9]
6-Z29 [0, 2, 3, 6, 7, 9]
6-30 [0, 1, 3, 6, 7, 9]
6-31 [0, 1, 4, 5, 7, 9]
6-32 [0, 2, 4, 5, 7, 9]
6-33 [0, 2, 3, 5, 7, 9]
6-34 [0, 1, 3, 5, 7, 9]
6-35 [0, 2, 4, 6, 8, 10]
6-Z36 [0, 1, 2, 3, 4, 7]
6-Z37 [0, 1, 2, 3, 4, 8]
6-Z38 [0, 1, 2, 3, 7, 8]
6-Z39 [0, 2, 3, 4, 5, 8]
6-Z40 [0, 1, 2, 3, 5, 8]
6-Z41 [0, 1, 2, 3, 6, 8]
6-Z42 [0, 1, 2, 3, 6, 9]
6-Z43 [0, 1, 2, 5, 6, 8]
6-Z44 [0, 1, 2, 5, 6, 9]
6-Z45 [0, 2, 3, 4, 6, 9]
6-Z46 [0, 1, 2, 4, 6, 9]
6-Z47 [0, 1, 2, 4, 7, 9]
6-Z48 [0, 1, 2, 5, 7, 9]
6-Z49 [0, 1, 3, 4, 7, 9]
6-Z50 [0, 1, 4, 6, 7, 9]
7-1 [0, 1, 2, 3, 4, 5, 6]
7-2 [0, 1, 2, 3, 4, 5, 7]
7-3 [0, 1, 2, 3, 4, 5, 8]
7-4 [0, 1, 2, 3, 4, 6, 7]
7-5 [0, 1, 2, 3, 5, 6, 7]
7-6 [0, 1, 2, 3, 4, 7, 8]
7-7 [0, 1, 2, 3, 6, 7, 8]
7-8 [0, 2, 3, 4, 5, 6, 8]
7-9 [0, 1, 2, 3, 4, 6, 8]
7-10 [0, 1, 2, 3, 4, 6, 9]
7-11 [0, 1, 3, 4, 5, 6, 8]
7-Z12 [0, 1, 2, 3, 4, 7, 9]
7-13 [0, 1, 2, 4, 5, 6, 8]
7-14 [0, 1, 2, 3, 5, 7, 8]
7-15 [0, 1, 2, 4, 6, 7, 8]
7-16 [0, 1, 2, 3, 5, 6, 9]
7-Z17 [0, 1, 2, 4, 5, 6, 9]
7-Z18 [0, 1, 4, 5, 6, 7, 9]
7-19 [0, 1, 2, 3, 6, 7, 9]
7-20 [0, 1, 2, 5, 6, 7, 9]
7-21 [0, 1, 2, 4, 5, 8, 9]
7-22 [0, 1, 2, 5, 6, 8, 9]
7-23 [0, 2, 3, 4, 5, 7, 9]
7-24 [0, 1, 2, 3, 5, 7, 9]
7-25 [0, 2, 3, 4, 6, 7, 9]
7-26 [0, 1, 3, 4, 5, 7, 9]
7-27 [0, 1, 2, 4, 5, 7, 9]
7-28 [0, 1, 3, 5, 6, 7, 9]
7-29 [0, 1, 2, 4, 6, 7, 9]

7-30 [0, 1, 2, 4, 6, 8, 9]
7-31 [0, 1, 3, 4, 6, 7, 9]
7-32 [0, 1, 3, 4, 6, 8, 9]
7-33 [0, 1, 2, 4, 6, 8, 10]
7-34 [0, 1, 3, 4, 6, 8, 10]
7-35 [0, 1, 3, 5, 6, 8, 10]
7-Z36 [0, 1, 2, 3, 5, 6, 8]
7-Z37 [0, 1, 3, 4, 5, 7, 8]
7-Z38 [0, 1, 2, 4, 5, 7, 8]
8-1 [0, 1, 2, 3, 4, 5, 6, 7]
8-2 [0, 1, 2, 3, 4, 5, 6, 8]
8-3 [0, 1, 2, 3, 4, 5, 6, 9]
8-4 [0, 1, 2, 3, 4, 5, 7, 8]
8-5 [0, 1, 2, 3, 4, 6, 7, 8]
8-6 [0, 1, 2, 3, 5, 6, 7, 8]
8-7 [0, 1, 2, 3, 4, 5, 8, 9]
8-8 [0, 1, 2, 3, 4, 7, 8, 9]
8-9 [0, 1, 2, 3, 6, 7, 8, 9]
8-10 [0, 2, 3, 4, 5, 6, 7, 9]
8-11 [0, 1, 2, 3, 4, 5, 7, 9]
8-12 [0, 1, 3, 4, 5, 6, 7, 9]
8-13 [0, 1, 2, 3, 4, 6, 7, 9]
8-14 [0, 1, 2, 4, 5, 6, 7, 9]
8-Z15 [0, 1, 2, 3, 4, 6, 8, 9]
8-16 [0, 1, 2, 3, 5, 7, 8, 9]
8-17 [0, 1, 3, 4, 5, 6, 8, 9]
8-18 [0, 1, 2, 3, 5, 6, 8, 9]
8-19 [0, 1, 2, 4, 5, 6, 8, 9]
8-20 [0, 1, 2, 4, 5, 7, 8, 9]
8-21 [0, 1, 2, 3, 4, 6, 8, 10]
8-22 [0, 1, 2, 3, 5, 6, 8, 10]
8-23 [0, 1, 2, 3, 5, 7, 8, 10]
8-24 [0, 1, 2, 4, 5, 6, 8, 10]
8-25 [0, 1, 2, 4, 6, 7, 8, 10]
8-26 [0, 1, 3, 4, 5, 7, 8, 10]
8-27 [0, 1, 2, 4, 5, 7, 8, 10]
8-28 [0, 1, 3, 4, 6, 7, 9, 10]
8-Z29 [0, 1, 2, 3, 5, 6, 7, 9]
9-1 [0, 1, 2, 3, 4, 5, 6, 7, 8]
9-2 [0, 1, 2, 3, 4, 5, 6, 7, 9]
9-3 [0, 1, 2, 3, 4, 5, 6, 8, 9]
9-4 [0, 1, 2, 3, 4, 5, 7, 8, 9]
9-5 [0, 1, 2, 3, 4, 6, 7, 8, 9]
9-6 [0, 1, 2, 3, 4, 5, 6, 8, 10]
9-7 [0, 1, 2, 3, 4, 5, 7, 8, 10]
9-8 [0, 1, 2, 3, 4, 6, 7, 8, 10]
9-9 [0, 1, 2, 3, 5, 6, 7, 8, 10]
9-10 [0, 1, 2, 3, 4, 6, 7, 9, 10]
9-11 [0, 1, 2, 3, 5, 6, 7, 9, 10]
9-12 [0, 1, 2, 4, 5, 6, 8, 9, 10]
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10-3 [0, 1, 2, 3, 4, 5, 6, 7, 9, 10]
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10-5 [0, 1, 2, 3, 4, 5, 7, 8, 9, 10]
10-6 [0, 1, 2, 3, 4, 6, 7, 8, 9, 10]
11-1 [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
12-1 [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

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